

Monthly Forecasts of Systematic Risk: An Evaluation

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Recent advances in covariance and variance estimators coupled with improvements in the quality of intra-day data have made possible more precise measurement of beta (systematic risk). In this paper we examine the forecastability of monthly betas for Dow Jones stocks. The out-of-sample forecasting exercise conducted in our study results in a dramatic reduction of forecast error of beta on average by over 80%, relative to the industry standard of the constant model. This finding has vast implications for all aspects of finance as precise forecasting of the beta parameter is of crucial importance.

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Recent advances in covariance and variance estimators coupled with improvements in the quality of intra-day data have made possible more precise measurement of beta (systematic risk). In this paper we examine the forecastability of monthly betas for Dow Jones stocks. The out-of-sample forecasting exercise conducted in our study results in a dramatic reduction of forecast error of beta on average by over 80%, relative to the industry standard of the constant model. This finding has vast implications for all aspects of finance as precise forecasting of the beta parameter is of crucial importance.

1. Introduction

Systematic risk as measured by beta is a foundation stone of modern finance theory and is a focal point of countless investment and financing decisions. The beta of a security represents its sensitivity to movements in the market. Precise forecasts of beta often on a monthly basis are of critical importance for asset pricing, cash flow valuation, risk management and performance evaluation. For example, fund managers often rebalance their portfolios on a monthly basis and require accurate forecasts of beta [Wang (2003) and Ghysels and Jacquier (2005)].

Much attention has been given to the time-varying nature of beta [Mandelker (1974), Keim and Stambaugh (1986), Ferson (1989) and Breen, Glosten and Jagannathan (1989)]. Ghysels (1998) examined various parametric time varying beta models and shows that no model can outperform the constant beta model. He examined models from Ferson (1989), Ferson and Harvey (1991, 1993) and Ferson and Korajczyk (1995) and Dumas and Solnik (1995).

This is the first out-of-sample forecast evaluation study to demonstrate a modelling approach that dramatically dominates the monthly constant model (the industry standard for over thirty years, following Fama and Macbeth, 1973). On average the reduction in forecast error is approximately 80%. In this study, out-of-sample beta forecasts based upon the constant model, based on a 5 year window of monthly returns (Fama and MacBeth, 1973) are evaluated against betas computed from 30 minute intra-day returns over a common forecast horizon of one month. This paper utilizes the approach of Barndorff-Nielsen and Shephard (2004) and Andersen, Bollerslev, Diebold and Wu (2006) to measure and forecast betas.

The realized beta measurement framework of Barndorff-Nielsen and Shephard (2004) and Andersen, Bollerslev, Diebold and Wu (2005 and 2006) follows on from the earlier related work on realized volatility of Merton (1980), French, Schwert and Stambaugh (1987) and Schwert (1989). A realized beta is the ratio of the stock and market return realized covariance and the market realized variance. Realized variance has been the focus of many recent studies such as Andersen and Bollerslev (1998), Andersen, Bollerslev, Diebold and Labys (2000, 2001, 2003) Andersen, Bollerslev, Diebold and Ebens (2001), Barndorff-Nielsen and Shephard (2001, 2002a, 2002b, 2004), Maheu and McCurdy (2002), Martens, van Dijk and Pooter (2004), Ghysels, Santa-Clara and Valkanov (2005), Koopman, Jungbacker and Hol (2005) that show that autoregressive time series models, computed on realized variance outperform popular models such as GARCH [Engle (1982) and Bollerslev (1986)].

In this paper we compute US monthly realized betas, using intraday, daily and monthly returns. We model and forecast realized betas with the constant, autoregressive and random walk models. Experimentation with in-sample estimation sizes of 60 and 36 months are conducted, with the forecast evaluation period being 34 months. This out-of-sample forecasting evaluation finds a dramatic reduction of forecast error of beta by up to 95%, relative to the industry standard of the constant model computed from monthly returns. On average over the Dow Jones stocks the forecast error reduction is approximately 80%.

This paper is organized as follows: Section 2 describes the data used in the study and section 3 outlines the beta measurement approach. The forecast evaluation is conducted in section 4 and the final section concludes the study.

2. Data

Monthly, daily and 30-minute intraday data are collected for the analysis. The data set is sourced from the Center for Research in Security Prices (CRSP) and Price-Data. Monthly and daily data of the companies are sourced from CRSP, while intraday and the Dow Jones Index data are from Price-Data. The data set begins on 1st October 1997 and continues until 31st December 2005.

Twenty-five companies in the Dow Jones Industrial Average (DJIA) and the DJIA Index are included in the data set. The DJIA is a price-weighted average of 30 companies and since 1928 it has been an extensively employed indicator of the stock market. Selecting companies from the Dow Jones Thirty Index provides a sufficient degree of liquidity. Initially the entire 30 companies of the DJIA were considered, however, due to the incompleteness of data, five companies were excluded from the sample.

3. Beta Measuring

Following the approach of Barndorff-Nielsen and Shephard (2004) and Andersen, Bollerslev, Diebold and Wu (2006), the logarithmic $N \times 1$ vector price process, p_t , is assumed to follow a multivariate continuous-time stochastic volatility diffusion,

$$dp_t = \mu_t dt + \Omega_t dW_t \quad (1)$$

where W_t is a standard N -dimensional Brownian motion process, Ω_t is the $N \times N$ positive definite diffusion matrix and μ_t is the N -dimensional instantaneous drift. Ω_t and μ_t are strictly stationary and jointly independent of W_t .

Defining the compounded Δ -period return as $r_{t+\Delta,\Delta} \equiv p_{t+\Delta} - p_t$, and following the theory of quadratic variation with the sampling frequency (Δ) tending to zero over the interval h , the following result is established:

$$\sum_{j=1, \dots, [h/\Delta]} r_{t+j\Delta, \Delta} \cdot r'_{t+j\Delta, \Delta} - \int_0^h \Omega_{t+\tau} d\tau \rightarrow 0 \quad (2)$$

See Barndorff-Nielsen and Shephard (2004) and Andersen, Bollerslev, Diebold and Wu (2006) for further details.

The beta of a security is the covariance of the security with the market divided by the variance of the market. The realized beta of a security is the realized covariance of a security and the market divided by the realized variance of the market. The realized covariance of a security i and the market M over a period $[t, t+h]$ is the sum of the product of the Δ -period returns of a security i and the market M , uniformly measured over the period $[t, t+h]$:

$$\hat{v}_{iM,t,t+h} = \sum_{j=1, \dots, [h/\Delta]} r_{i,t+j\Delta, \Delta} \cdot r_{M,t+j\Delta, \Delta} \quad (3)$$

The realized variance over a period $[t, t+h]$ is the sum of the squared Δ -period returns of the market M uniformly measured over the period $[t, t+h]$:

$$\hat{v}_{M,t,t+h}^2 = \sum_{j=1, \dots, [h/\Delta]} r_{M,t+j\Delta, \Delta}^2 \quad (4)$$

and the realized beta is the realized covariance of the security and the market divided by the realized variance of the market:

$$\hat{\beta}_{i,t,t+h} = \frac{\hat{v}_{iM,t,t+h}}{\hat{v}_{M,t,t+h}^2} = \frac{\sum_{j=1, \dots, [h/\Delta]} r_{i,t+j,\Delta,\Delta} \cdot r_{M,t+j,\Delta,\Delta}}{\sum_{j=1, \dots, [h/\Delta]} r_{M,t+j,\Delta,\Delta}^2} \quad (5)$$

In this study realized monthly betas are computed using 30-minutes returns, hence h is one month and the Δ -period is 30-minutes. The 30-minute interval creates a balance between measurement error and market microstructure noise effects and mitigates the need for filtering the realized betas. The market trades from 9.30am to 4.00pm. There are thirteen 30-minutes interval returns for each day and consequently approximately 285 underlying observations for each month. Figure 1 presents the monthly realized betas for US companies and Figures 2 and 3 display the autocorrelation and partial autocorrelation functions. In general, these functions display a decay in the autocorrelation and a cut-off in the partial autocorrelation at an early lag, indicating low order autoregressive processes.

Commonly used in financial practice and research is the rolling estimator of beta based upon a 5 year window of monthly returns (as above with h set at 5 years and Δ equal to 1 month). In addition we also compute realized betas when h is set to 5 years and Δ equal to 1 day. We repeat the above with a shorter estimation window of h set at 3 years and Δ set at 1 month and 1 day, respectively.

4. Forecast Evaluation

Month-ahead forecasting of beta is conducted for each of the US stocks over a forecast evaluation period from March 2003 until December 2005. For the autoregressive models, a low order process, as suggested by the ACFs and PACFs [Figures 2 and 3], is fitted to the time series of monthly realized betas. The coefficients are then used to forecast the next one-month ahead beta. The following equation is the forecasting equation for the autoregressive model with p lags (AR(p)).

$$\beta_{t+1} = \alpha_0 + \alpha_1\beta_t + \alpha_2\beta_{t-1} + \dots + \alpha_p\beta_{t-(p-1)} \quad (7)$$

where α_p are coefficients from performing an OLS regression on the following equation

$$\beta_t = \alpha_0 + \alpha_1\beta_{t-1} + \alpha_2\beta_{t-2} + \dots + \alpha_p\beta_{t-p} \quad (8)$$

In addition, a variety of constant models are examined; the 5 year Benchmark Monthly model, the 5 year Benchmark Daily model, the 3 year Benchmark Monthly model, the 3 year Benchmark Daily model, the 5 year Constant model and the 3 year Constant model.

The 5 year Benchmark Monthly model uses monthly returns from the previous 60 months to compute a realized beta that represents the one-month ahead forecast following Fama and MacBeth (1973). The 5 year Benchmark Daily model uses daily returns from the previous 60 months to compute a realized beta that represents the one-month ahead forecast. The 3 year Benchmark Monthly model uses monthly returns from the previous 36 months to compute a realized beta that represents the one-month ahead forecast. The 3 year Benchmark Daily model uses daily returns from the previous 36 months to compute a realized beta that represents the one-month ahead forecast.

The 5 year Constant model takes the average of the previous 60 monthly realized betas computed with 30 minute interval returns as the one-month ahead forecast. The 3 year Constant model takes the average of the previous 36 monthly realized betas computed with 30 minute interval returns as the one-month ahead forecast.

The random walk model approach uses the current monthly beta computed with 30 minute interval returns as the one-month ahead forecast.

$$\beta_{t+1} = \beta_t \quad (9)$$

The out-of-sample beta forecasts are made over the identical period to the constant and autoregressive models. To test the forecasting ability of each approach, the mean squared error (MSE) and mean absolute error (MAE) for each company is computed. The MSE and MAE are calculated as follows:

$$MSE = \frac{1}{m} \sum_{j=1}^m (\beta_j - \hat{\beta}_j)^2 \quad (10)$$

$$MAE = \frac{1}{m} \sum_{j=1}^m |\beta_j - \hat{\beta}_j| \quad (11)$$

where m is the number of months in the out-of-sample evaluation, and β_j is the realized beta at month j and $\hat{\beta}_j$ is the corresponding forecast, for each model. In out-of-sample forecast evaluations in which the return sampling frequency is not sufficiently high relative to the beta

measurement frequency, it is recommended that the β_j measurement is Kalman filtered as suggested by Andersen, Bollerslev, Diebold and Wu (2005).

To provide a comprehensible evaluation of the beta forecast a series of tables are assembled. Table 2 and Table 3 provide the MSE and MAE of the one-month-ahead forecast of US betas based upon the previous 60 months, respectively. Table 4 and Table 5 provide the MSE and MAE of the one-month-ahead forecast of US betas based upon the previous 36 months, respectively. Table 6 and Table 7 provide the average error of the one-month-ahead forecast of US Betas based on the previous 60 and 36 months, respectively, the average is computed by taking the mean of the 25 companies in the sample. To provide a detailed analysis on the reduction when using the AR(3) comparative to using the Benchmark Monthly, Benchmark Daily and Constant models, Table 8 and Table 9 present the relative and absolute reductions of the MSE and MAE for each company based on the previous 60 months, respectively. Table 10 and Table 11 show the reduction in MSE and MAE that an AR(3) would offer based on 36 months, respectively, relative to the Benchmark Monthly, Benchmark Daily and Constant models. Table 8 to Table 9 is constructed using the data from Table 2 to Table 5.

Firstly, the average error of one-month-ahead forecast of US betas based on the previous 60 and 36 months obtainable in Table 6 and Table 7, respectively will be interpreted. Secondly, a detailed examination of specific companies is conducted; this involves an inspection of Table 8 to Table 11.

In absolute terms the AR(3) produced the best result based upon 60 months and the AR(1) produced the best result based upon 36 months and overall. The results based upon 36 months for the AR(1), AR(2) and AR(3) models are marginally different. The AR(3) model is favoured because the 3 lags assist to diminish the impact of potential outliers. The average error of one-month-ahead forecasts depict that beta forecasts using the autoregressive model with 3 lags based upon an in-sample size of 60 or 36 months produce the optimal results relative to the other models. On average the autoregressive model with 3 lags based on 60 or 36 months of realized betas, computed from 30 minute returns reduces MSE and MAE of the commonly used rolling estimator of beta based upon a 5 year window of monthly returns (the 5 year Benchmark Monthly model) by approximately 80% and 60% (Table 6 and Table 7), respectively. Table 6 and Table 7 show that the autoregressive models consistently outperform the other models in both in-sample sizes of 60 and 36 months.

The average reduction in MSE and MAE when using the AR(3) model based upon the previous 60 months relative to the random walk model is 25% and 14%, respectively, results for 36 months show a further reduction of 26% and 15%. For the 60 and 36 months in-sample size, 23 of the 25 companies had a random walk forecast that resulted in a higher MSE or MAE than the AR(3) model (for 2 companies, i.e. Altria Group INC and American Express where RW dominated the AR(1) model, the improvements were only marginal). Results show using the AR(3) model to forecast betas is superior to the random walk. It is expected that the random walk model would perform poorly, as betas do not appear to have a unit root, however, the model performed reasonably well for some companies.

In Table 8 and Table 9, the performance of the autoregressive model with 3 lags based upon the previous 60 months relative to the other non-autoregressive models is evaluated. The autoregressive model often shows substantial reductions in MSE and MAE values. First, the rolling estimator of beta based upon a 5 year window of monthly returns (5 year Benchmark Monthly model) showed JP Morgan Chase's MSE and MAE reduce by 95% and 82% in relative terms and by 0.551 and 0.616 in absolute terms, Alcoa INC's MSE and MAE reduce by 94%(0.806) and 79%(0.714) and Intel Corporation reduce by 93%(0.710) and 77%(0.635), respectively. Secondly, the rolling estimator of beta based upon a 5 year window of daily returns (5 year Benchmark Daily model) showed JP Morgan Chase's MSE and MAE reduce by 93%(0.418) and 80%(0.521), Citigroup INC's MSE and MAE reduce by 92%(0.351) and 77%(0.468) and General Electric by 88%(0.248) and 71%(0.362), respectively. Finally, the constant model showed American International Group's MSE and MAE reduced by 61%(0.030) and 41%(0.075), JP Morgan Chase's MSE and MAE reduced by 55%(0.038) and 43%(0.099) and Microsoft Corporation's MSE and MAE reduce by 44%(0.032) and 34%(0.020), respectively. This shows that that the non-autoregressive models perform very poorly with some companies, relative to the AR(3) model.

Though the non-autoregressive models overall poorly against the autoregressive models, there were a few companies using the non-autoregressive models that performed better than the AR(3) model (the results of these companies are bolded in Table 8 and Table 9). The maximum increase in MSE and MAE when using the AR(3) model against other models is inspected. Firstly, the 5 year Benchmark Monthly model results showed the MSE and MAE for American International Group increase by 6% and 9% in relative terms and by 0.001 and 0.009 in absolute

terms and for Altria Group INC an increase of 5%(0.002) and 11%(0.017), respectively, when using the AR(3) model. Secondly, the 5 year Benchmark Daily model showed the MSE and MAE for Johnson and Johnson increase by 35%(0.008) and 17%(0.023), and for Merck and Co an increase of 16% (0.011) and 16%(0.030), respectively. Finally, the constant results showed the MSE and MAE for United Technology Corporation an increase of 8%(0.004) and 15%(0.023), and for Home Depot INC an increase of 7% (0.002) and 23%(0.007), respectively. Results from Table 10 and Table 11 show very similar results to Table 8 and Table 9, showing the consistency and robustness of the results.

It is seen that, 15 of the 25 companies had their MSE more than halved, when using the AR(3) model opposed to both the commonly used rolling estimator of beta based upon a 5 year window of monthly and daily returns (refer to Table 8). 17 of the 25 companies had their MSE more than halved, when using the AR(3) model as opposed to both the commonly used rolling estimator of beta based upon a 3 year window of monthly and daily returns (refer to Table 10).

The substantial improvements in beta forecasts are of immense importance to finance practice, in areas such as portfolio construction and asset valuations where accurate beta forecast are very important [Wang (2003) and Ghysels and Jacquier (2005)]. There are only a few companies where non-autoregressive models perform better than the AR(3) model. The forecasts of systematic risk using an autoregressive model with 3 lags based upon an in-sample size of 60 or 36 months of realized betas computed from 30 minute returns on average reduces the MSE and MAE of beta forecasts by approximately 80% and 60%, respectively, compared to the 5 year Benchmark Monthly model.

5. Conclusion

This paper was motivated by the demand by academics and practitioners alike for precise forecasts of monthly systematic risk as measured by beta. Coupling the state of the art financial econometric technique of estimating realized covariances and variances with the availability of superior quality high frequency data, we demonstrate models that produce forecast errors dramatically less than the standard benchmark monthly models which have been the industry standard for over 30 years, following Fama and MacBeth (1973).

The forecasts of systematic risk using an autoregressive model with 3 lags based upon an in-sample size of 60 or 36 months of realized betas computed from 30 minute returns produce the optimal results in terms of reducing forecast error relative to the other models. In general, the reduction of MSE and MAE was by approximately 80% and 60%, respectively, compared to the 5 year Benchmark Monthly model.

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Table 1: Sample of US Companies

	Ticker	Company
1	MMM	3M CO
2	AA	ALCOA INC
3	MO	ALTRIA GROUP INC
4	AIG	AMER INTL GRP
5	AXP	AMERICAN EXPRESS
6	BA	BOEING CO
7	CAT	CATERPILLAR INC
8	C	CITIGROUP INC
9	KO	COCA-COLA CO
10	DIS	DISNEY (WALT) CO
11	DD	DU PONT (EI)
12	GE	GENERAL ELECTRIC
13	GM	GENERAL MOTORS
14	HPQ	HEWLETT-PACKARD
15	HD	HOME DEPOT INC
16	IBM	IBM
17	INTC	INTEL CORP
18	JNJ	JOHNSON&JOHNSON
19	JPM	JP MORGAN CHASE
20	MCD	MCDONALDS CORP
21	MRK	MERCK & CO
22	MSFT	MICROSOFT CORP
23	PFE	PFIZER
24	UTX	UNITED TECH CORP
25	WMT	WAL-MART STORES

**Table 2: MSE of One-Month-Ahead Forecast of US Betas
based on the previous 60 months**

Company	Benchmark Monthly	Benchmark Daily	Constant	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)	RW
3M CO	0.024	0.036	0.022	0.016	0.015	0.016	0.016	0.017	0.018
ALCOA INC	0.861	0.134	0.075	0.059	0.053	0.055	0.056	0.057	0.079
ALTRIA GROUP INC	0.039	0.052	0.054	0.041	0.041	0.041	0.041	0.042	0.040
AMER INTL GRP	0.018	0.042	0.049	0.025	0.021	0.019	0.019	0.018	0.025
AMERICAN EXPRESS	0.309	0.468	0.105	0.067	0.064	0.068	0.077	0.087	0.056
BOEING CO	0.049	0.053	0.055	0.041	0.040	0.038	0.036	0.037	0.055
CATERPILLAR INC	0.103	0.039	0.050	0.047	0.040	0.041	0.039	0.041	0.064
CITIGROUP INC	0.320	0.381	0.053	0.037	0.031	0.030	0.030	0.034	0.036
COCA-COLA CO	0.060	0.017	0.021	0.021	0.021	0.020	0.021	0.022	0.034
DISNEY (WALT) CO	0.217	0.198	0.038	0.037	0.039	0.038	0.038	0.038	0.055
DU PONT (EI)	0.083	0.059	0.040	0.033	0.034	0.035	0.035	0.035	0.044
GENERAL ELECTRIC	0.050	0.283	0.037	0.036	0.036	0.035	0.036	0.041	0.038
GENERAL MOTORS	0.164	0.111	0.078	0.063	0.059	0.059	0.058	0.059	0.067
HEWLETT -PACKARD	0.491	0.309	0.064	0.051	0.056	0.052	0.053	0.050	0.070
HOME DEPOT INC	0.132	0.199	0.029	0.029	0.030	0.031	0.032	0.034	0.054
IBM	0.443	0.152	0.032	0.024	0.025	0.023	0.023	0.023	0.031
INTEL CORP	0.765	0.345	0.068	0.060	0.063	0.055	0.059	0.060	0.075
JOHNSON &JOHNSON	0.077	0.025	0.032	0.034	0.035	0.033	0.033	0.032	0.058
JP MORGAN CHASE	0.582	0.449	0.069	0.032	0.033	0.031	0.032	0.031	0.034
MCDONALDS CORP	0.054	0.073	0.091	0.053	0.051	0.049	0.049	0.049	0.054
MERCK & CO	0.089	0.067	0.075	0.074	0.077	0.078	0.075	0.078	0.110
MICROSOFT CORP	0.356	0.285	0.073	0.054	0.046	0.041	0.044	0.047	0.043
PFIZER	0.087	0.049	0.050	0.048	0.052	0.052	0.053	0.057	0.085
UNITED TECH CORP	0.181	0.154	0.053	0.053	0.054	0.057	0.059	0.059	0.102
WAL-MART STORES	0.045	0.103	0.042	0.041	0.033	0.035	0.038	0.039	0.049

The Benchmark Monthly model uses monthly returns from the previous 60 months to compute a realized beta that represents the one-month ahead forecast. The Benchmark Daily model uses daily returns from the previous 60 months to compute a realized beta that represents the one-month ahead forecast. The Constant model takes the average of the previous 60 monthly realized betas computed with 30 minute interval returns as the one-month ahead forecast. For the autoregressive models, a low order process, is fitted to the previous 60 monthly realized betas computed with 30 minute interval returns. The coefficients are then used to forecast the next one-month ahead beta. The random walk model approach uses the current monthly beta computed with 30 minute interval returns as the one-month ahead forecast. The forecast evaluation period covers the period from 2003:3 through 2005:12. Minimum value for each company is bolded.

**Table 3: MAE of One-Month-Ahead Forecast of US Betas
based on the previous 60 months**

Company	Benchmark Monthly	Benchmark Daily	Constant Intraday	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)	RW
3M CO	0.126	0.156	0.126	0.104	0.101	0.104	0.103	0.105	0.105
ALCOA INC	0.901	0.288	0.238	0.198	0.186	0.187	0.193	0.192	0.217
ALTRIA GROUP INC	0.154	0.197	0.200	0.168	0.169	0.171	0.167	0.168	0.154
AMER INTL GRP	0.098	0.185	0.182	0.115	0.111	0.107	0.106	0.103	0.126
AMERICAN EXPRESS	0.518	0.652	0.262	0.205	0.201	0.209	0.222	0.237	0.189
BOEING CO	0.168	0.180	0.193	0.164	0.160	0.143	0.142	0.145	0.185
CATERPILLAR INC	0.285	0.159	0.182	0.178	0.162	0.157	0.155	0.159	0.210
CITIGROUP INC	0.551	0.605	0.201	0.155	0.138	0.137	0.137	0.146	0.152
COCA-COLA CO	0.212	0.114	0.125	0.123	0.125	0.122	0.127	0.126	0.153
DISNEY (WALT) CO	0.420	0.393	0.169	0.159	0.166	0.160	0.159	0.160	0.185
DU PONT (EI)	0.247	0.208	0.149	0.140	0.143	0.149	0.150	0.153	0.176
GENERAL ELECTRIC	0.179	0.510	0.157	0.157	0.154	0.148	0.151	0.163	0.149
GENERAL MOTORS	0.366	0.288	0.193	0.178	0.172	0.162	0.161	0.165	0.186
HEWLETT -PACKARD	0.646	0.514	0.200	0.179	0.188	0.178	0.179	0.171	0.223
HOME DEPOT INC	0.314	0.412	0.144	0.142	0.145	0.147	0.151	0.157	0.204
IBM	0.651	0.366	0.146	0.129	0.133	0.126	0.124	0.124	0.135
INTEL CORP	0.824	0.545	0.227	0.207	0.210	0.189	0.198	0.202	0.214
JOHNSON &JOHNSON	0.235	0.134	0.150	0.154	0.157	0.157	0.156	0.151	0.193
JP MORGAN CHASE	0.747	0.652	0.230	0.141	0.139	0.131	0.133	0.131	0.152
MCDONALDS CORP	0.189	0.223	0.246	0.180	0.174	0.173	0.172	0.170	0.174
MERCK & CO	0.231	0.185	0.201	0.207	0.212	0.215	0.209	0.215	0.263
MICROSOFT CORP	0.570	0.502	0.235	0.199	0.178	0.165	0.168	0.178	0.161
PFIZER	0.234	0.178	0.158	0.154	0.159	0.159	0.161	0.168	0.234
UNITED TECH CORP	0.373	0.362	0.151	0.159	0.165	0.174	0.178	0.176	0.232
WAL-MART STORES	0.175	0.277	0.160	0.159	0.148	0.153	0.160	0.161	0.186

The Benchmark Monthly model uses monthly returns from the previous 60 months to compute a realized beta that represents the one-month ahead forecast. The Benchmark Daily model uses daily returns from the previous 60 months to compute a realized beta that represents the one-month ahead forecast. The Constant model takes the average of the previous 60 monthly realized betas computed with 30 minute interval returns as the one-month ahead forecast. For the autoregressive models, a low order process, is fitted to the previous 60 monthly realized betas computed with 30 minute interval returns. The coefficients are then used to forecast the next one-month ahead beta. The random walk model approach uses the current monthly beta computed with 30 minute interval returns as the one-month ahead forecast. The forecast evaluation period covers the period from 2003:3 through 2005:12. Minimum value for each company is bolded.

**Table 4: MSE of One-Month-Ahead Forecast of US Betas
based on the previous 36 months**

Company	Benchmark Monthly	Benchmark Daily	Constant Intraday	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)	RW
3M CO	0.037	0.039	0.022	0.015	0.016	0.017	0.017	0.019	0.018
ALCOA INC	1.026	0.224	0.054	0.053	0.051	0.053	0.055	0.057	0.079
ALTRIA GROUP INC	0.152	0.048	0.048	0.036	0.038	0.038	0.039	0.042	0.040
AMER INTL GRP	0.187	0.109	0.042	0.021	0.019	0.018	0.020	0.021	0.025
AMERICAN EXPRESS	0.332	0.339	0.086	0.062	0.061	0.065	0.065	0.072	0.056
BOEING CO	0.164	0.081	0.034	0.035	0.038	0.040	0.040	0.045	0.055
CATERPILLAR INC	0.155	0.052	0.035	0.037	0.035	0.035	0.037	0.039	0.064
CITIGROUP INC	0.230	0.296	0.041	0.032	0.028	0.027	0.027	0.025	0.036
COCA-COLA CO	0.079	0.017	0.023	0.022	0.023	0.022	0.023	0.024	0.034
DISNEY (WALT) CO	0.285	0.371	0.047	0.042	0.042	0.040	0.042	0.044	0.055
DU PONT (EI)	0.098	0.070	0.036	0.034	0.036	0.038	0.042	0.042	0.044
GENERAL ELECTRIC	0.032	0.240	0.034	0.035	0.036	0.037	0.039	0.043	0.038
GENERAL MOTORS	0.172	0.165	0.065	0.063	0.061	0.063	0.061	0.063	0.067
HEWLETT -PACKARD	0.838	0.288	0.046	0.044	0.047	0.048	0.055	0.053	0.070
HOME DEPOT INC	0.222	0.113	0.030	0.033	0.036	0.037	0.039	0.038	0.054
IBM	0.569	0.146	0.030	0.027	0.028	0.026	0.027	0.028	0.031
INTEL CORP	0.962	0.336	0.063	0.064	0.072	0.061	0.068	0.071	0.075
JOHNSON &JOHNSON	0.094	0.033	0.033	0.035	0.039	0.039	0.037	0.037	0.058
JP MORGAN CHASE	0.595	0.414	0.062	0.032	0.034	0.031	0.032	0.033	0.034
MCDONALDS CORP	0.206	0.055	0.073	0.048	0.051	0.046	0.047	0.046	0.054
MERCK & CO	0.117	0.105	0.078	0.078	0.080	0.075	0.072	0.077	0.110
MICROSOFT CORP	0.144	0.254	0.058	0.045	0.042	0.038	0.041	0.049	0.043
PFIZER	0.048	0.090	0.046	0.046	0.044	0.045	0.048	0.052	0.085
UNITED TECH CORP	0.119	0.154	0.052	0.053	0.057	0.064	0.067	0.070	0.102
WAL-MART STORES	0.069	0.051	0.032	0.032	0.031	0.034	0.037	0.040	0.049

The Benchmark Monthly model uses monthly returns from the previous 36 months to compute a realized beta that represents the one-month ahead forecast. The Benchmark Daily model uses daily returns from the previous 36 months to compute a realized beta that represents the one-month ahead forecast. The Constant model takes the average of the previous 36 monthly realized betas computed with 30 minute interval returns as the one-month ahead forecast. For the autoregressive models, a low order process, is fitted to the previous 36 monthly realized betas computed with 30 minute interval returns. The coefficients are then used to forecast the next one-month ahead beta. The random walk model approach uses the current monthly beta computed with 30 minute interval returns as the one-month ahead forecast. The forecast evaluation period covers the period from 2003:3 through 2005:12. Minimum value for each company is bolded.

**Table 5: MAE of One-Month-Ahead Forecast of US Betas
based on the previous 36 months**

Company	Benchmark Monthly	Benchmark Daily	Constant Intraday	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)	RW
3M CO	0.154	0.160	0.128	0.100	0.103	0.107	0.107	0.113	0.105
ALCOA INC	0.977	0.408	0.204	0.191	0.190	0.192	0.196	0.198	0.217
ALTRIA GROUP INC	0.307	0.175	0.186	0.152	0.156	0.155	0.160	0.166	0.154
AMER INTL GRP	0.366	0.289	0.159	0.115	0.111	0.106	0.109	0.113	0.126
AMERICAN EXPRESS	0.511	0.530	0.235	0.196	0.195	0.201	0.197	0.208	0.189
BOEING CO	0.314	0.234	0.147	0.151	0.159	0.156	0.156	0.166	0.185
CATERPILLAR INC	0.351	0.190	0.151	0.154	0.150	0.145	0.146	0.151	0.210
CITIGROUP INC	0.441	0.525	0.168	0.142	0.131	0.124	0.128	0.126	0.152
COCA-COLA CO	0.229	0.108	0.129	0.127	0.128	0.125	0.124	0.127	0.153
DISNEY (WALT) CO	0.499	0.578	0.174	0.167	0.164	0.158	0.162	0.168	0.185
DU PONT (EI)	0.274	0.224	0.143	0.138	0.146	0.152	0.160	0.162	0.176
GENERAL ELECTRIC	0.132	0.463	0.141	0.143	0.146	0.150	0.154	0.160	0.149
GENERAL MOTORS	0.353	0.369	0.168	0.171	0.172	0.174	0.177	0.183	0.186
HEWLETT-PACKARD	0.865	0.497	0.159	0.162	0.170	0.173	0.175	0.174	0.223
HOME DEPOT INC	0.420	0.286	0.147	0.153	0.156	0.161	0.168	0.162	0.204
IBM	0.680	0.341	0.138	0.131	0.135	0.128	0.130	0.135	0.135
INTEL CORP	0.943	0.537	0.204	0.204	0.218	0.199	0.207	0.213	0.214
JOHNSON & JOHNSON	0.256	0.149	0.146	0.159	0.171	0.173	0.165	0.156	0.193
JP MORGAN CHASE	0.722	0.613	0.205	0.139	0.141	0.136	0.142	0.144	0.152
MCDONALDS CORP	0.398	0.184	0.216	0.162	0.164	0.159	0.165	0.167	0.174
MERCK & CO	0.254	0.256	0.213	0.216	0.217	0.213	0.203	0.216	0.263
MICROSOFT CORP	0.332	0.479	0.203	0.172	0.166	0.156	0.158	0.176	0.161
PFIZER	0.166	0.255	0.155	0.153	0.156	0.155	0.158	0.167	0.234
UNITED TECH CORP	0.285	0.342	0.146	0.152	0.165	0.179	0.185	0.189	0.232
WAL-MART STORES	0.226	0.179	0.140	0.143	0.149	0.157	0.162	0.168	0.186

The Benchmark Monthly model uses monthly returns from the previous 36 months to compute a realized beta that represents the one-month ahead forecast. The Benchmark Daily model uses daily returns from the previous 36 months to compute a realized beta that represents the one-month ahead forecast. The Constant model takes the average of the previous 36 monthly realized betas computed with 30 minute interval returns as the one-month ahead forecast. For the autoregressive models, a low order process, is fitted to the previous 36 monthly realized betas computed with 30 minute interval returns. The coefficients are then used to forecast the next one-month ahead beta. The random walk model approach uses the current monthly beta computed with 30 minute interval returns as the one-month ahead forecast. The forecast evaluation period covers the period from 2003:3 through 2005:12. Minimum value for each company is bolded.

**Table 6: Average Error of One-Month-Ahead Forecast of US Betas
based on the previous 60 months**

Company	Benchmark Monthly	Benchmark Daily	Constant	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)	RW
MSE	0.2239	0.1634	0.0542	0.0430	0.0420	0.0413	0.0421	0.0435	0.0551
MAE	0.3767	0.3314	0.1850	0.1622	0.1598	0.1569	0.1585	0.1610	0.1823

The Benchmark Monthly model uses monthly returns from the previous 60 months to compute a realized beta that represents the one-month ahead forecast. The Benchmark Daily model uses daily returns from the previous 60 months to compute a realized beta that represents the one-month ahead forecast. The Constant model takes the average of the previous 60 monthly realized betas computed with 30 minute interval returns as the one-month ahead forecast. For the autoregressive models, a low order process, is fitted to the previous 60 monthly realized betas computed with 30 minute interval returns. The coefficients are then used to forecast the next one-month ahead beta. The random walk model approach uses the current monthly beta computed with 30 minute interval returns as the one-month ahead forecast. Minimum values are bolded. Average values are calculated by taking the mean of the 25 companies in the sample. The forecast evaluation period covers the period from 2003:3 through 2005:12.

**Table 7: Average Error of One-Month-Ahead Forecast of US Betas
based on the previous 36 months**

Company	Benchmark Monthly	Benchmark Daily	Constant	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)	RW
MSE	0.2773	0.1636	0.0468	0.0410	0.0418	0.0415	0.0431	0.0452	0.0551
MAE	0.4181	0.3314	0.1682	0.1557	0.1584	0.1574	0.1598	0.1643	0.1823

The Benchmark Monthly model uses monthly returns from the previous 36 months to compute a realized beta that represents the one-month ahead forecast. The Benchmark Daily model uses daily returns from the previous 36 months to compute a realized beta that represents the one-month ahead forecast. The Constant model takes the average of the previous 36 monthly realized betas computed with 30 minute interval returns as the one-month ahead forecast. For the autoregressive models, a low order process, is fitted to the previous 36 monthly realized betas computed with 30 minute interval returns. The coefficients are then used to forecast the next one-month ahead beta. The random walk model approach uses the current monthly beta computed with 30 minute interval returns as the one-month ahead forecast. Minimum values are bolded. Average values are calculated by taking the mean of the 25 companies in the sample. The forecast evaluation period covers the period from 2003:3 through 2005:12.

Table 8: AR3 Vs. Constant Models (MSE)
(Reduction in MSE when using AR3 - Relative and Absolute values)
based on the previous 60 months

	Relative			Absolute		
	Benchmark Monthly	Benchmark Daily	Constant	Benchmark Monthly	Benchmark Daily	Constant
3M CO	35%	56%	27%	0.008	0.020	0.006
ALCOA INC	94%	59%	27%	0.806	0.079	0.020
ALTRIA GROUP INC	-5%	21%	24%	-0.002	0.011	0.013
AMER INTL GRP	-6%	55%	61%	-0.001	0.023	0.030
AMERICAN EXPRESS	78%	85%	35%	0.241	0.400	0.037
BOEING CO	23%	29%	31%	0.011	0.015	0.017
CATERPILLAR INC	60%	-4%	18%	0.062	-0.002	0.009
CITIGROUP INC	91%	92%	43%	0.290	0.351	0.023
COCA-COLA CO	67%	-16%	5%	0.040	-0.003	0.001
DISNEY (WALT) CO	82%	81%	0%	0.179	0.160	0.000
DU PONT (EI)	58%	41%	13%	0.048	0.024	0.005
GENERAL ELECTRIC	30%	88%	5%	0.015	0.248	0.002
GENERAL MOTORS	64%	47%	24%	0.105	0.052	0.019
HEWLETT-PACKARD	89%	83%	19%	0.439	0.257	0.012
HOME DEPOT INC	76%	84%	-7%	0.101	0.168	-0.002
IBM	95%	85%	28%	0.420	0.129	0.009
INTEL CORP	93%	84%	19%	0.710	0.290	0.013
JOHNSON&JOHNSON	57%	-35%	-3%	0.044	-0.008	-0.001
JP MORGAN CHASE	95%	93%	55%	0.551	0.418	0.038
MCDONALDS CORP	8%	33%	46%	0.005	0.024	0.042
MERCK & CO	12%	-16%	-4%	0.011	-0.011	-0.003
MICROSOFT CORP	88%	86%	44%	0.315	0.244	0.032
PFIZER	40%	-7%	-4%	0.035	-0.003	-0.002
UNITED TECH CORP	69%	63%	-8%	0.124	0.097	-0.004
WAL-MART STORES	22%	66%	17%	0.010	0.068	0.007
Max[^]	95%	93%	61%	0.806	0.418	0.042
Min^{^^}	-6%	-35%	-8%	-0.002	-0.011	-0.004
>20%[*]	21	20	12			
>50%^{**}	16	15	2			
>80%^{***}	8	10	0			

[^]Maximum value in the column

^{^^}Minimum value in the column

^{*} number of values over 20% in the column

^{**} number of values over 50% in the column

^{***} number of values over 80% in the column

The Benchmark Monthly model uses monthly returns from the previous 60 months to compute a realized beta that represents the one-month ahead forecast. The Benchmark Daily model uses daily returns from the previous 60 months to compute a realized beta that represents the one-month ahead forecast. The Constant model takes the average of the previous 60 monthly realized betas computed with 30 minute interval returns as the one-month ahead forecast. The AR(3) model is fitted to the previous 60 monthly realized betas computed with 30 minute interval returns. The coefficients are then used to forecast the next one-month ahead beta.

Relative values are the percentage reduction in MSE measurements when comparing one-month-ahead forecast methods of the benchmark monthly, benchmark daily and constant model with the AR(3) model.

Absolute values are the absolute reduction in MSE measurements when comparing one-month-ahead forecast methods of the benchmark monthly, benchmark daily and constant model with the AR(3) model.

The forecast evaluation period covers the period from 2003:3 through 2005:12. The bolded percentages indicate they are negative.

Table 9: AR3 Vs. Constant Models (MAE)
(Reduction in MAE when using AR3 - Relative and Absolute values)
based on the previous 60 months

	Relative			Absolute		
	Benchmark Monthly	Benchmark Daily	Constant	Benchmark Monthly	Benchmark Daily	Constant
3M CO	17%	34%	17%	0.022	0.052	0.022
ALCOA INC	79%	35%	21%	0.714	0.101	0.051
ALTRIA GROUP INC	-11%	13%	15%	-0.017	0.026	0.029
AMER INTL GRP	-9%	42%	41%	-0.009	0.078	0.075
AMERICAN EXPRESS	60%	68%	20%	0.309	0.443	0.053
BOEING CO	15%	21%	26%	0.025	0.037	0.050
CATERPILLAR INC	45%	2%	14%	0.128	0.002	0.025
CITIGROUP INC	75%	77%	32%	0.414	0.468	0.064
COCA-COLA CO	42%	-7%	2%	0.090	-0.008	0.003
DISNEY (WALT) CO	62%	59%	5%	0.260	0.233	0.009
DU PONT (EI)	40%	28%	0%	0.098	0.059	0.000
GENERAL ELECTRIC	17%	71%	6%	0.031	0.362	0.009
GENERAL MOTORS	56%	44%	16%	0.204	0.126	0.031
HEWLETT-PACKARD	72%	65%	11%	0.468	0.336	0.022
HOME DEPOT INC	53%	64%	-2%	0.167	0.265	-0.003
IBM	81%	66%	14%	0.525	0.240	0.020
INTEL CORP	77%	65%	17%	0.635	0.356	0.038
JOHNSON&JOHNSON	33%	-17%	-5%	0.078	-0.023	-0.007
JP MORGAN CHASE	82%	80%	43%	0.616	0.521	0.099
MCDONALDS CORP	9%	22%	30%	0.016	0.050	0.073
MERCK & CO	7%	-16%	-7%	0.016	-0.030	-0.014
MICROSOFT CORP	71%	67%	30%	0.405	0.337	0.070
PFIZER	32%	10%	-1%	0.075	0.019	-0.001
UNITED TECH CORP	53%	52%	-15%	0.199	0.188	-0.023
WAL-MART STORES	13%	45%	4%	0.022	0.124	0.007
Max^	82%	80%	43%	0.714	0.521	0.099
Min^^	-11%	-17%	-15%	-0.017	-0.030	-0.023
>20%*	17	18	8			
>50%**	12	11	0			
>80%***	2	0	0			

^Maximum value in the column

^^Minimum value in the column

* number of values over 20% in the column

** number of values over 50% in the column

*** number of values over 80% in the column

The Benchmark Monthly model uses monthly returns from the previous 60 months to compute a realized beta that represents the one-month ahead forecast. The Benchmark Daily model uses daily returns from the previous 60 months to compute a realized beta that represents the one-month ahead forecast. The Constant model takes the average of the previous 60 monthly realized betas computed with 30 minute interval returns as the one-month ahead forecast. The AR(3) model is fitted to the previous 60 monthly realized betas computed with 30 minute interval returns. The coefficients are then used to forecast the next one-month ahead beta.

Relative values are the percentage reduction in MAE measurements when comparing one-month-ahead forecast methods of the benchmark monthly, benchmark daily and constant model with the AR(3) model.

Absolute values are the absolute reduction in MAE measurements when comparing one-month-ahead forecast methods of the benchmark monthly, benchmark daily and constant model with the AR(3) model.

The forecast evaluation period covers the period from 2003:3 through 2005:12. The bolded percentages indicate they are negative.

Table 10: AR3 Vs. Constant Models (MSE)
(Reduction in MSE when using AR3 - Relative and Absolute values)
based on the previous 36 months

	Relative			Absolute		
	Benchmark Monthly	Benchmark Daily	Constant	Benchmark Monthly	Benchmark Daily	Constant
3M CO	54%	57%	23%	0.020	0.022	0.005
ALCOA INC	95%	76%	2%	0.973	0.171	0.001
ALTRIA GROUP INC	75%	20%	21%	0.114	0.010	0.010
AMER INTL GRP	90%	83%	57%	0.169	0.091	0.024
AMERICAN EXPRESS	80%	81%	24%	0.267	0.274	0.021
BOEING CO	76%	51%	-18%	0.124	0.041	-0.006
CATERPILLAR INC	77%	33%	0%	0.120	0.017	0.000
CITIGROUP INC	88%	91%	34%	0.203	0.269	0.014
COCA-COLA CO	72%	-27%	4%	0.057	-0.005	0.001
DISNEY (WALT) CO	86%	89%	15%	0.245	0.331	0.007
DU PONT (EI)	61%	45%	-6%	0.060	0.032	-0.002
GENERAL ELECTRIC	-17%	85%	-9%	-0.005	0.203	-0.003
GENERAL MOTORS	63%	62%	3%	0.109	0.102	0.002
HEWLETT-PACKARD	94%	83%	-4%	0.790	0.240	-0.002
HOME DEPOT INC	83%	67%	-23%	0.185	0.076	-0.007
IBM	95%	82%	13%	0.543	0.120	0.004
INTEL CORP	94%	82%	3%	0.901	0.275	0.002
JOHNSON&JOHNSON	58%	-19%	-18%	0.055	-0.006	-0.006
JP MORGAN CHASE	95%	93%	50%	0.564	0.383	0.031
MCDONALDS CORP	78%	16%	37%	0.160	0.009	0.027
MERCK & CO	36%	29%	4%	0.042	0.030	0.003
MICROSOFT CORP	74%	85%	34%	0.106	0.216	0.020
PFIZER	6%	50%	2%	0.003	0.045	0.001
UNITED TECH CORP	46%	59%	-23%	0.055	0.090	-0.012
WAL-MART STORES	51%	34%	-6%	0.035	0.017	-0.002
Max[^]	95%	93%	57%	0.973	0.383	0.031
Min^{^^}	-17%	-27%	-23%	-0.005	-0.006	-0.012
>20%*	23	22	8			
>50%**	21	17	1			
>80%***	10	10	0			

[^]Maximum value in the column

^{^^}Minimum value in the column

* number of values over 20% in the column

** number of values over 50% in the column

*** number of values over 80% in the column

The Benchmark Monthly model uses monthly returns from the previous 36 months to compute a realized beta that represents the one-month ahead forecast. The Benchmark Daily model uses daily returns from the previous 36 months to compute a realized beta that represents the one-month ahead forecast. The Constant model takes the average of the previous 36 monthly realized betas computed with 30 minute interval returns as the one-month ahead forecast. The AR(3) is fitted to the previous 36 monthly realized betas computed with 30 minute interval returns. The coefficients are then used to forecast the next one-month ahead beta.

Relative values are the percentage reduction in MSE measurements when comparing one-month-ahead forecast methods of the benchmark monthly, benchmark daily and constant model with the AR(3) model.

Absolute values are the absolute reduction in MSE measurements when comparing one-month-ahead forecast methods of the benchmark monthly, benchmark daily and constant model with the AR(3) model.

The forecast evaluation period covers the period from 2003:3 through 2005:12. The bolded percentages indicate they are negative.

Table 11: AR3 Vs. Constant Models (MAE)
(Reduction in MAE when using AR3 - Relative and Absolute values)
based on the previous 36 months

	Relative			Absolute		
	Benchmark Monthly	Benchmark Daily	Constant	Benchmark Monthly	Benchmark Daily	Constant
3M CO	30%	33%	16%	0.047	0.049	0.021
ALCOA INC	80%	53%	6%	0.785	0.096	0.012
ALTRIA GROUP INC	49%	12%	17%	0.152	0.042	0.031
AMER INTL GRP	71%	63%	33%	0.260	0.079	0.053
AMERICAN EXPRESS	61%	62%	14%	0.310	0.451	0.034
BOEING CO	50%	33%	-6%	0.158	0.024	-0.009
CATERPILLAR INC	59%	23%	4%	0.206	0.014	0.006
CITIGROUP INC	72%	76%	26%	0.317	0.481	0.044
COCA-COLA CO	45%	-16%	3%	0.104	-0.011	0.004
DISNEY (WALT) CO	68%	73%	9%	0.341	0.235	0.016
DU PONT (EI)	44%	32%	-6%	0.122	0.056	-0.009
GENERAL ELECTRIC	-14%	68%	-6%	-0.018	0.360	-0.009
GENERAL MOTORS	51%	53%	-4%	0.179	0.114	-0.006
HEWLETT-PACKARD	80%	65%	-9%	0.692	0.341	-0.014
HOME DEPOT INC	62%	44%	-10%	0.259	0.251	-0.014
IBM	81%	62%	7%	0.552	0.238	0.010
INTEL CORP	79%	63%	2%	0.744	0.346	0.005
JOHNSON&JOHNSON	32%	-16%	-18%	0.083	-0.039	-0.027
JP MORGAN CHASE	81%	78%	34%	0.586	0.516	0.069
MCDONALDS CORP	60%	14%	26%	0.239	0.064	0.057
MERCK & CO	16%	17%	0%	0.041	-0.028	0.000
MICROSOFT CORP	53%	67%	23%	0.176	0.346	0.047
PFIZER	7%	39%	0%	0.011	0.023	0.000
UNITED TECH CORP	37%	48%	-23%	0.106	0.183	-0.033
WAL-MART STORES	30%	12%	-12%	0.069	0.120	-0.017
Max[^]	81%	80%	34%	0.785	0.516	0.069
Min^{^^}	-14%	-29%	-23%	-0.018	-0.039	-0.033
>20%*	21	18	5			
>50%**	15	11	0			
>80%***	3	0	0			

[^]Maximum value in the column

^{^^}Minimum value in the column

* number of values over 20% in the column

** number of values over 50% in the column

*** number of values over 80% in the column

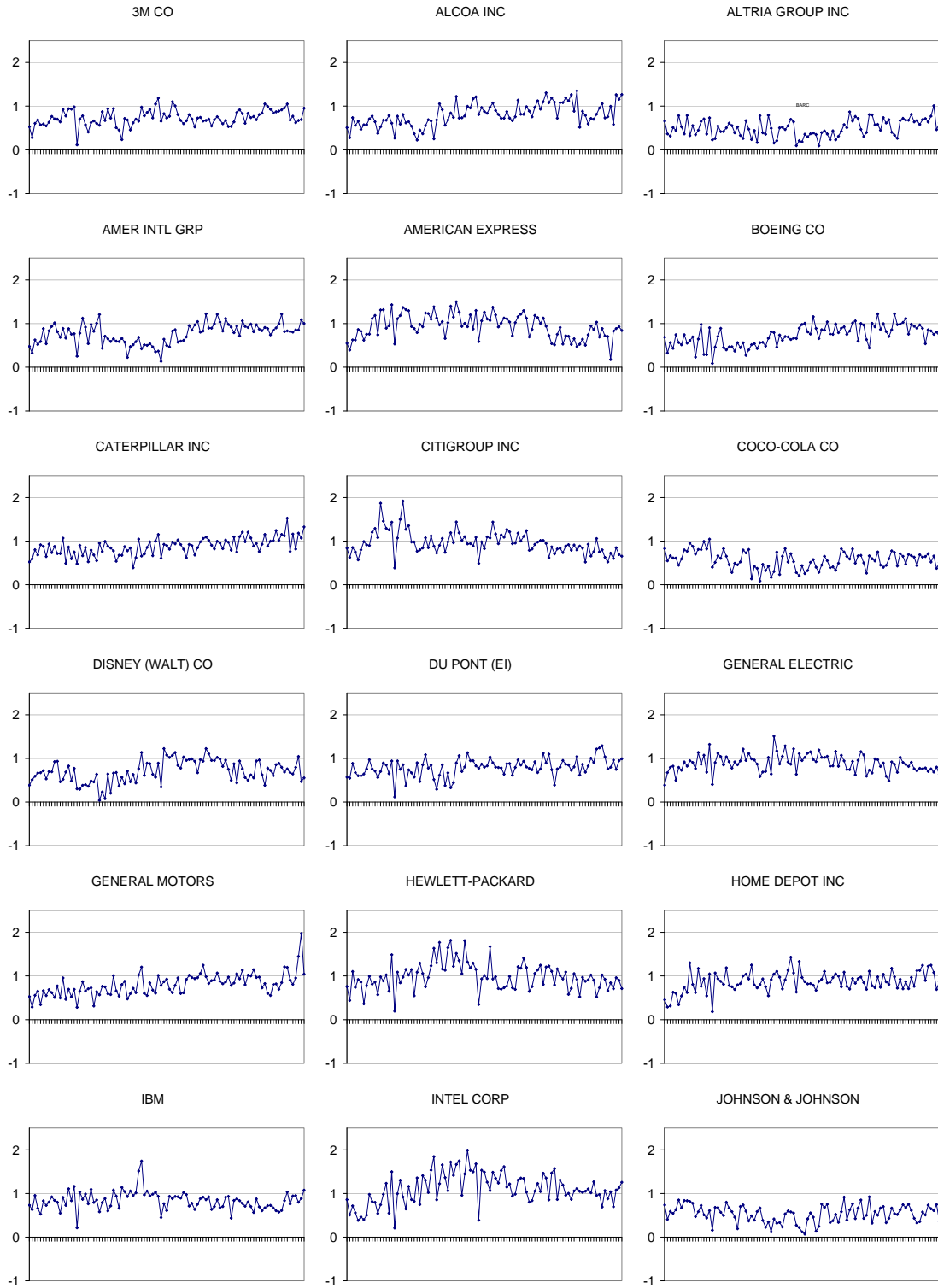
The Benchmark Monthly model uses monthly returns from the previous 36 months to compute a realized beta that represents the one-month ahead forecast. The Benchmark Daily model uses daily returns from the previous 36 months to compute a realized beta that represents the one-month ahead forecast. The Constant model takes the average of the previous 36 monthly realized betas computed with 30 minute interval returns as the one-month ahead forecast. The AR(3) model is fitted to the previous 36 monthly realized betas computed with 30 minute interval returns. The coefficients are then used to forecast the next one-month ahead beta.

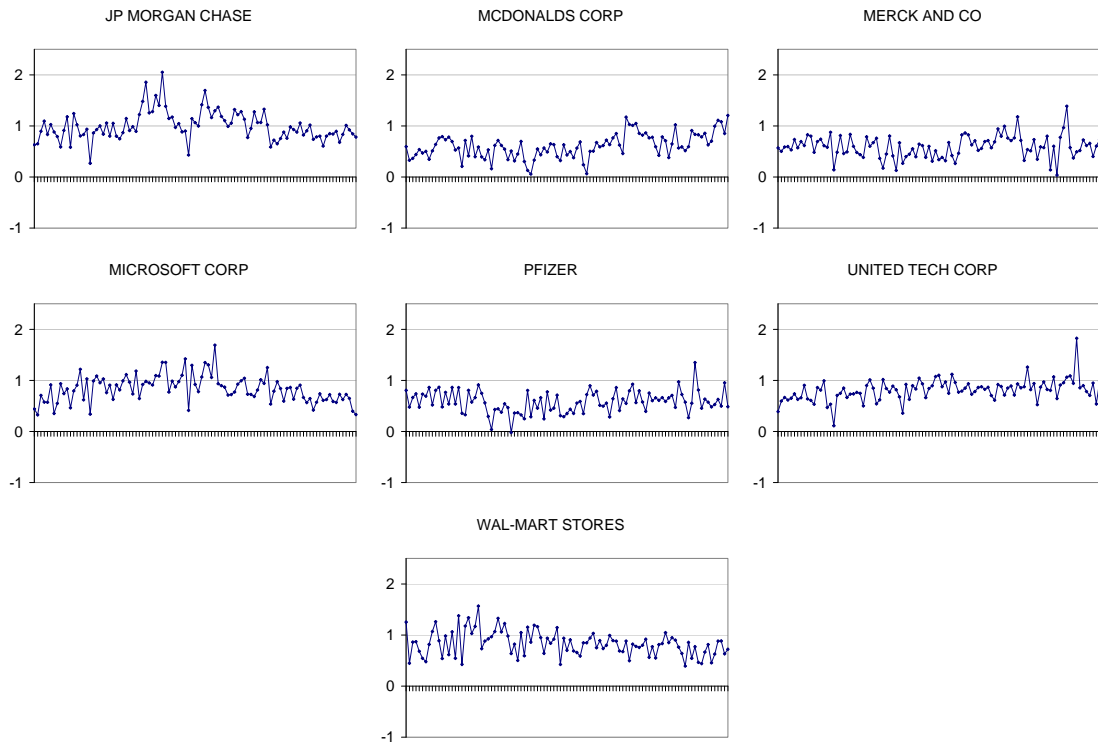
Relative values are the percentage reduction in MAE measurements when comparing one-month-ahead forecast methods of the benchmark monthly, benchmark daily and constant model with the AR(3) model.

Absolute values are the absolute reduction in MAE measurements when comparing one-month-ahead forecast methods of the benchmark monthly, benchmark daily and constant model with the AR(3) model.

The forecast evaluation period covers the period from 2003:3 through 2005:12. The bolded percentages indicate they are negative.

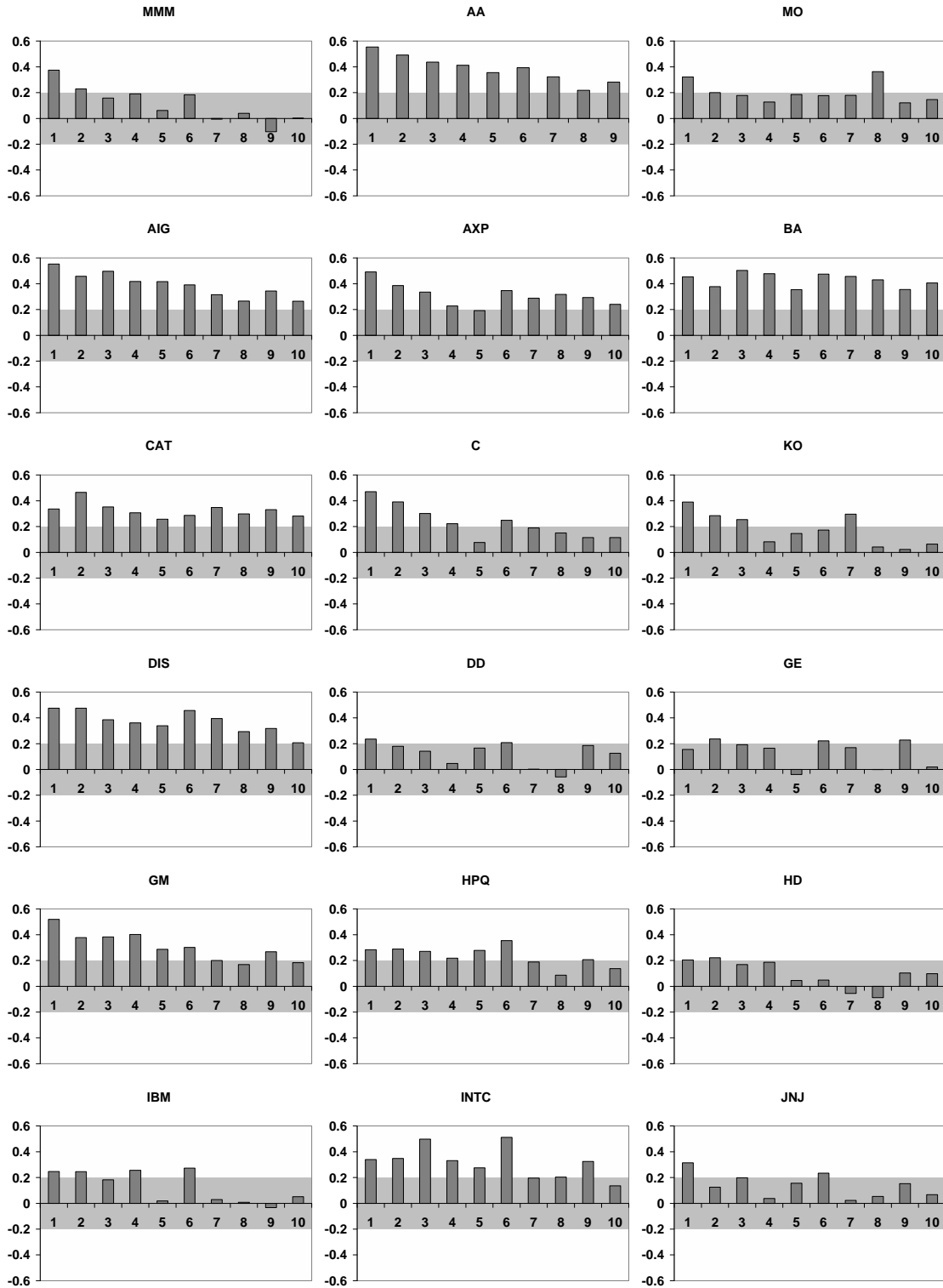
Figure 1: Monthly Realized Betas for US Companies

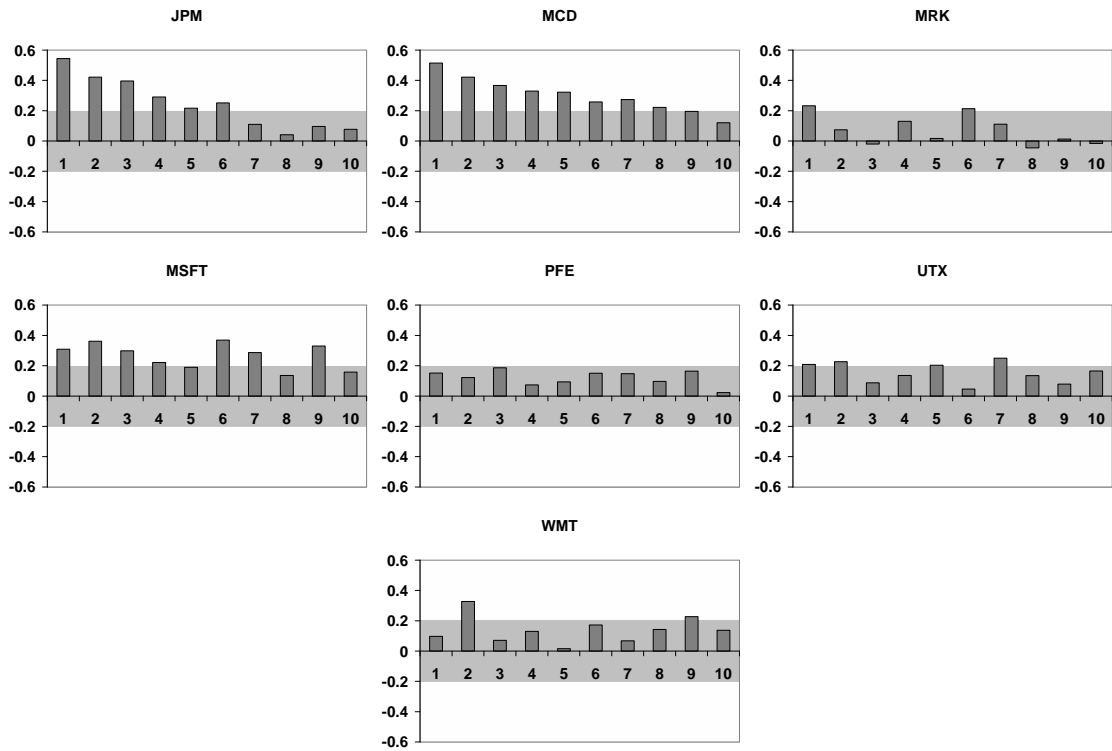




Realized betas are computed from 30 minute returns and the sample covers the period from 1997:10 through to 2005:12.

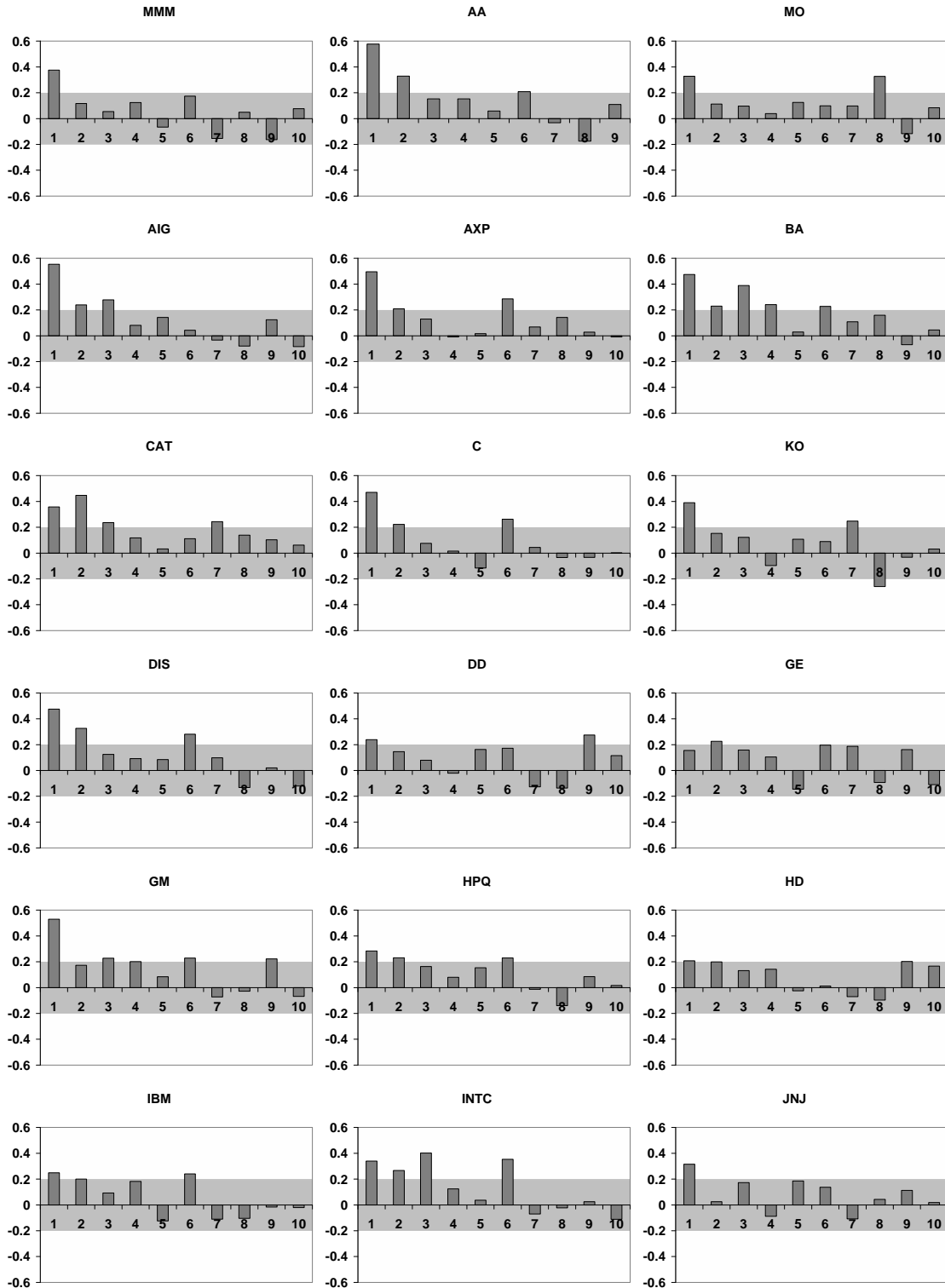
Figure 2: Autocorrelation Functions for Realized Betas of US Companies

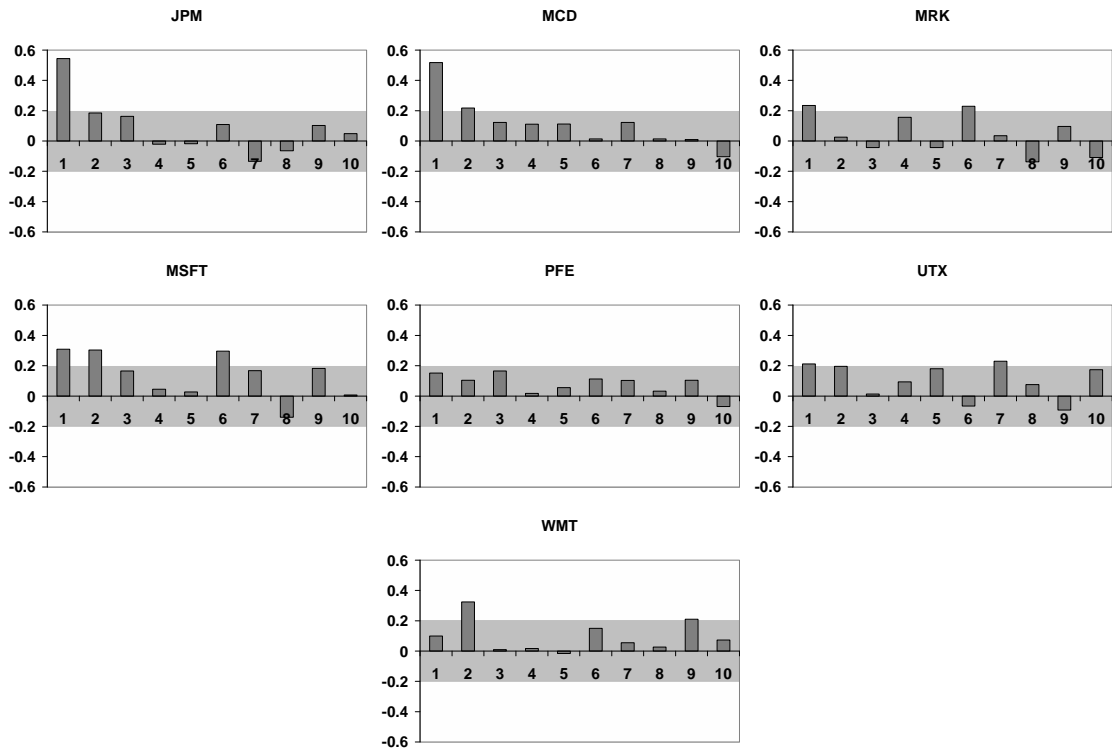




The sample period covers the period from 1997:10 through 2005:12.

Figure 3: Partial Autocorrelation Functions for Realized Betas of US Companies





The sample period covers the period from 1997:10 through 2005:12.