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VaR-implied correlations

Paul Kofman
The University of Melbourne

Rachel Campbell
The University of Maastricht

Motivation

After a decade of deriving *VaRs* for ever more complicated non-normal, non-linear asset payoffs, more attention has recently been paid to practical implementation issues, e.g., the portfolio implications.

In particular, market lore suggests that extreme price changes coincide more frequently than they ought to...

Sullivan (1995, *RISK*); Blyth (1996, *RISK*);

Longin & Solnik (1995, JIMF); Karolyi and Stulz (1996, *JF*)

What are the market risk implications?

What are the portfolio allocation implications?

Is there supporting empirical evidence?

Does correlation vary (significantly) with the size of returns?

Correlation models

Unconditional Correlation Measures

(fat-tailed) multivariate distributions
extreme value theory
copulas

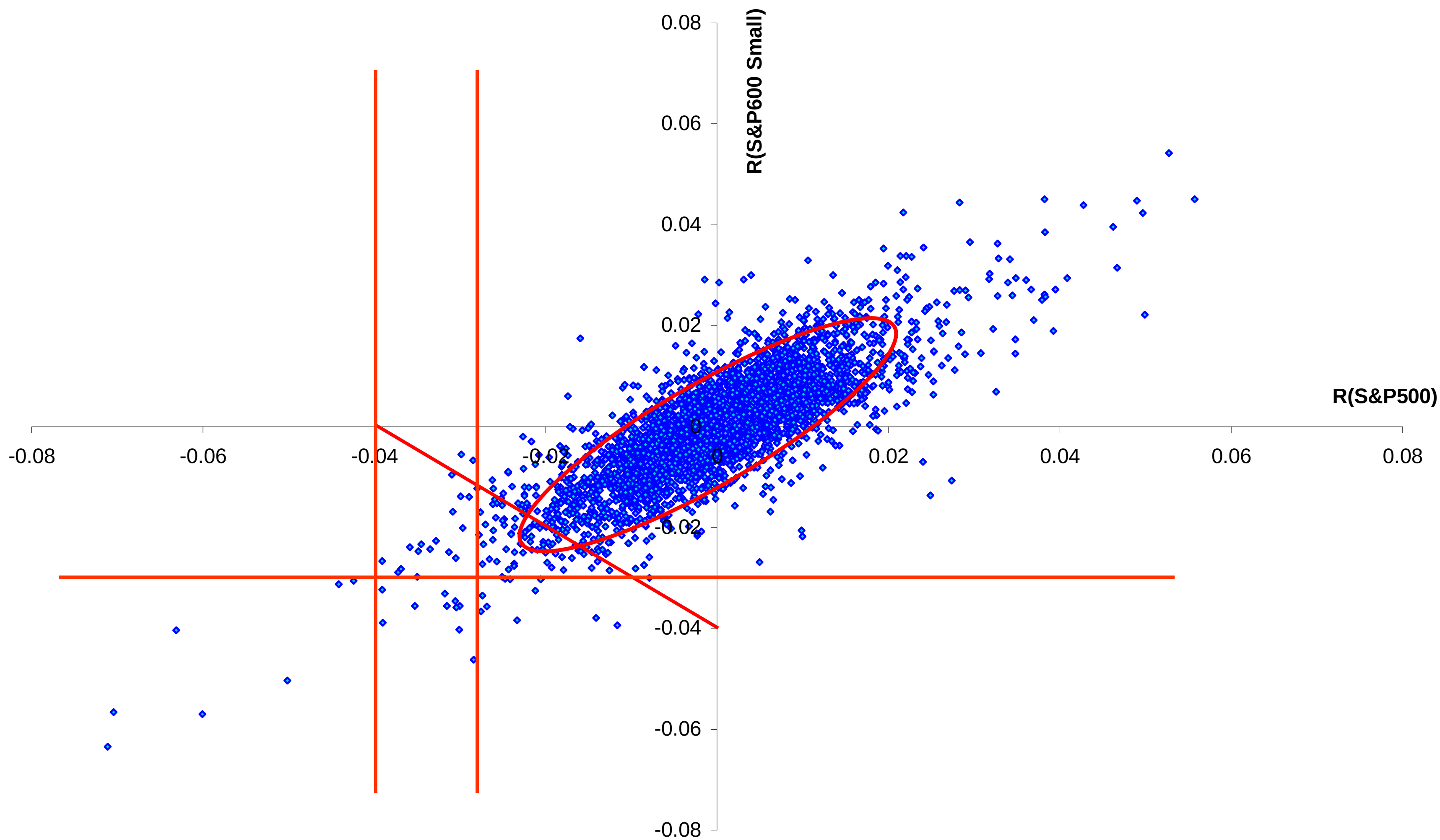
Conditional Measures

time-varying – BEKK, DCC
size-dependent – truncated, censored, elliptical, EVA, copulas

Implied Correlation Structure

rainbow options
*VaR*s (Campbell, Koedijk and Kofman, *FAJ*, 2002)

Characterizing correlations



VaR-implied correlations

Tail estimators are not always useful for portfolio allocation purposes, but consider the tail quantile:

$$q_c = \zeta_c \sigma$$

the *VaR* quantile return that will not be exceeded with 100(1-*c*)% probability

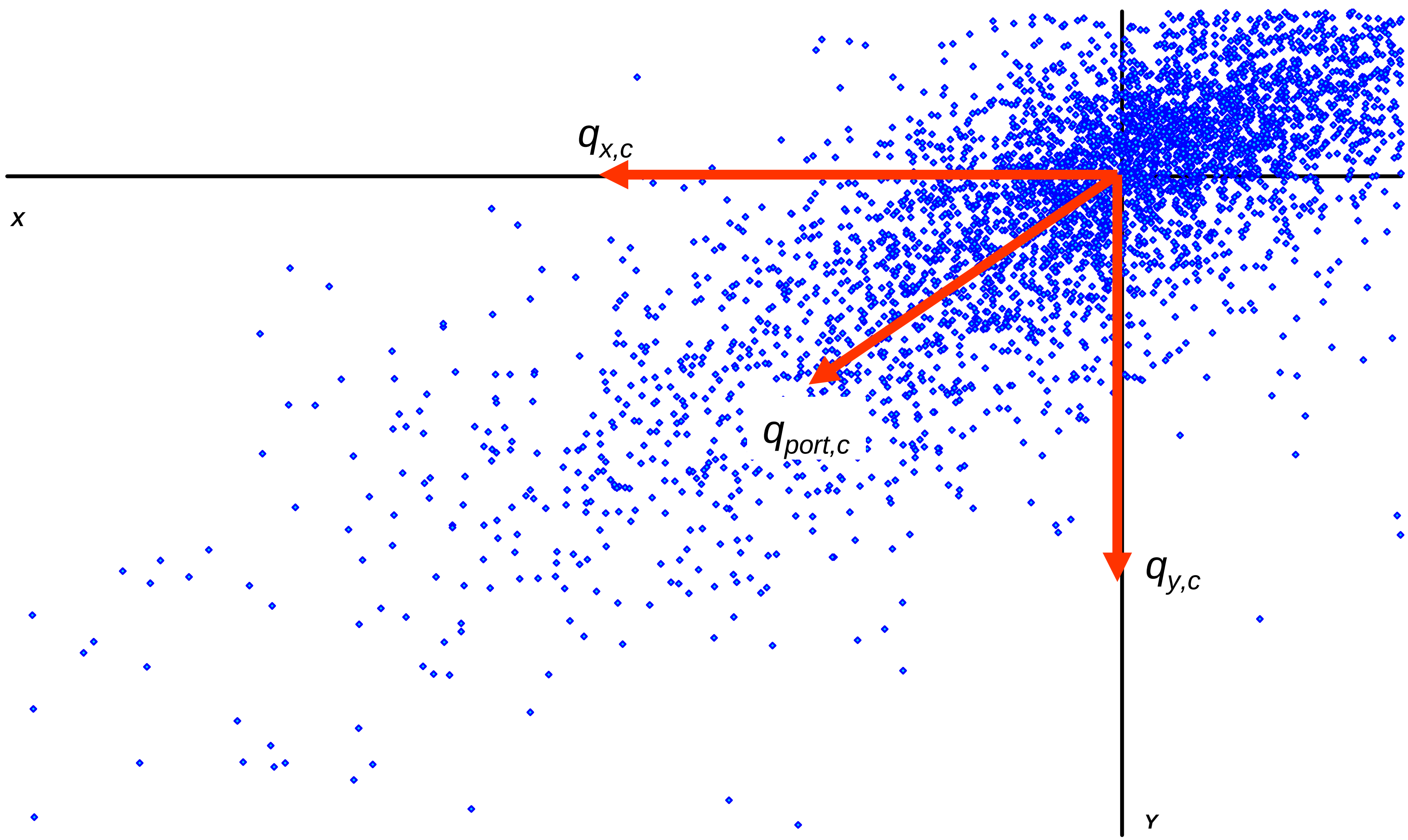
Now rewrite in terms of portfolio variance

$$q_{port,c}^2 = \zeta_c^2 \left(w_x^2 \sigma_x^2 + w_y^2 \sigma_y^2 + 2 w_x w_y \sigma_{xy} \right)$$

and substitute the portfolio component *VaR*s

$$\rho_c = \frac{q_{port,c}^2 - w_x^2 q_{x,c}^2 - w_x^2 q_{y,c}^2}{2 w_x w_y q_{x,c} q_{y,c}}$$

VaR-implied correlations



VaR-implied correlations

Jorion (1996) gives the standard error for quantile *i*, conditional on a normal distribution

$$se(q_i s) = q_i \times se(s \mid \phi)$$

$$se(s \mid \phi) = \sqrt{\left(\frac{1}{2T}\right)}$$

but our quantile correlation estimator is a ratio of squared quantiles

$$g_i'(\mu) = \partial g(x) / \partial x_i$$

$$g(x) = g(\mu) + \sum_{i=1}^{m} g_i'(\mu)(x_i - \mu) + o\left(n^{-1}\right)$$

$$\operatorname{var}(g(x)) = \sum_{i=1}^{m} \left(g_i'(\mu)\right)^2 \operatorname{var}(x_i) + \sum_{i \neq} \sum_{j=1}^{m} g_i'(\mu) g_j'(\mu) \operatorname{cov}\left(x_i, x_j\right) + o\left(n^{-1}\right)$$

VaR-implied correlations

which gives

$$\operatorname{var}\left(\rho_c\right)=A^2\operatorname{var}\left(q_{port,c}\right)+\left[\frac{w_x}{w_yE\left(q_{y,c}\right)}+\frac{B}{2C}\right]^2\operatorname{var}\left(q_{x,c}\right)+\left[\frac{w_y}{w_xE\left(q_{x,c}\right)}+\frac{B}{2D}\right]^2\operatorname{var}\left(q_{y,c}\right)\ldots$$

$$-2A\left[\frac{w_x}{w_yE\left(q_{y,c}\right)}+\frac{B}{2C}\right]\operatorname{cov}\left(q_{port,c},q_{x,c}\right)-2A\left[\frac{w_y}{w_xE\left(q_{x,c}\right)}+\frac{B}{2D}\right]\operatorname{cov}\left(q_{port,c},q_{y,c}\right)\ldots$$

$$+2\left[\frac{w_x}{w_yE\left(q_{y,c}\right)}+\frac{B}{2C}\right]\left[\frac{w_y}{w_xE\left(q_{x,c}\right)}+\frac{B}{2D}\right]\operatorname{cov}\left(q_{x,c},q_{y,c}\right)$$

where

$$A=\frac{E\left(q_{port,c}\right)}{w_xE\left(q_{x,c}\right)w_yE\left(q_{y,c}\right)} \qquad B=E^2\left(q_{port,c}\right)-w_x^2E^2\left(q_{x,c}\right)-w_y^2E^2\left(q_{y,c}\right)$$

$$C=w_xE^2\left(q_{x,c}\right)w_yE\left(q_{y,c}\right) \qquad D=w_xE\left(q_{x,c}\right)w_yE^2\left(q_{y,c}\right)$$

$$\operatorname{var}\left(q_c\right)=\frac{c\left(1-c\right)}{n\phi\left(q_c\right)^2} \qquad \operatorname{cov}\left(q_{x,c},q_{y,c}\right)=\frac{c\left(1-c\right)}{n\phi\left(q_{x.c}\right)\phi\left(q_{y,c}\right)}$$

Empirical implementation

Many conditional correlation applications use international stock index returns.

We take a domestic portfolio perspective for a US investor.

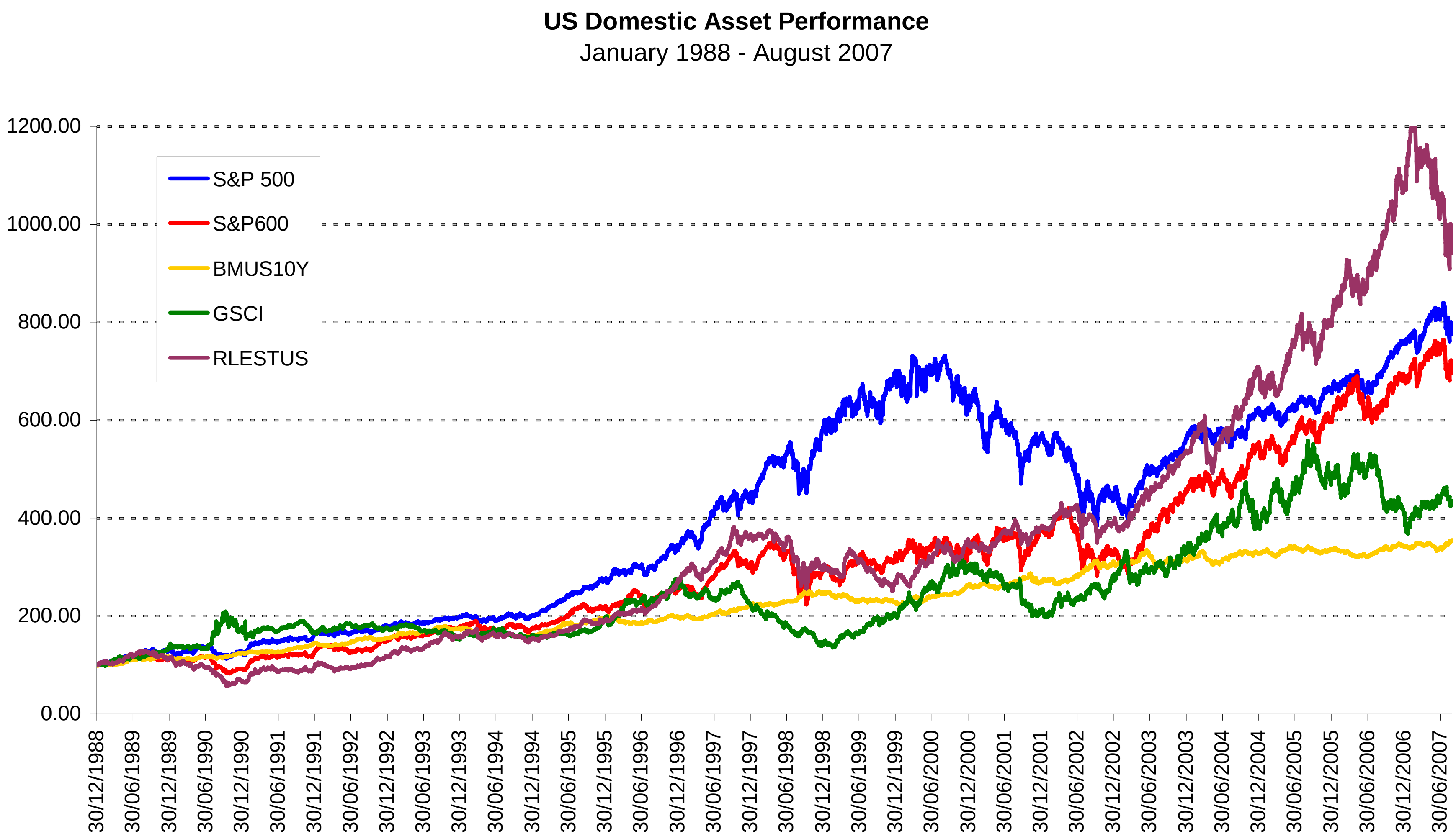
Our dataset contains daily (total) returns for the

S&P500, S&P600, USCORPB, USGB10YR, GSCI, USREI

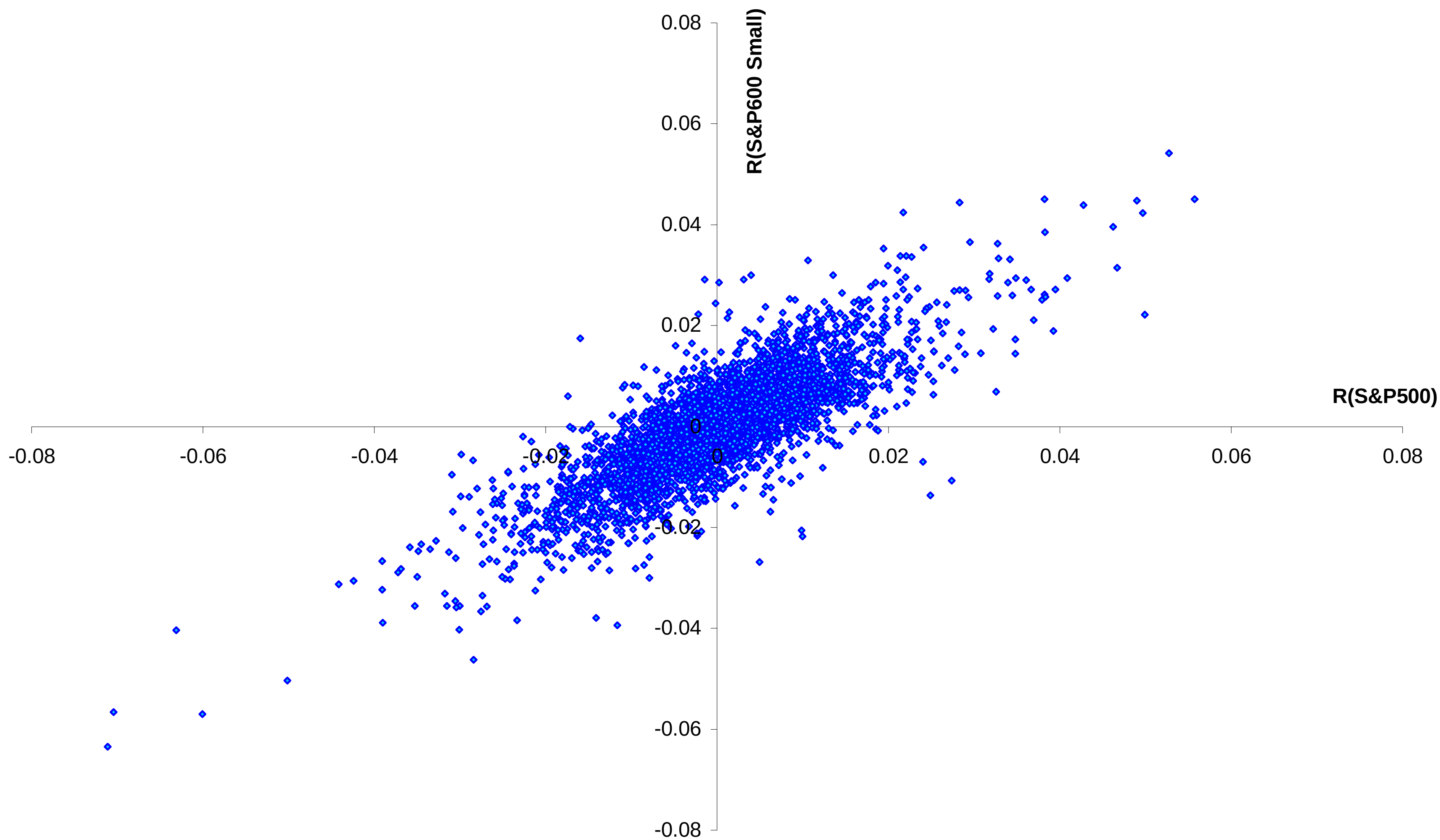
for the period Jan-1989 until Sep-2007 (4867 observations).

We standardize the returns with an AR(1)-GARCH(1,1) filter for the log-returns.

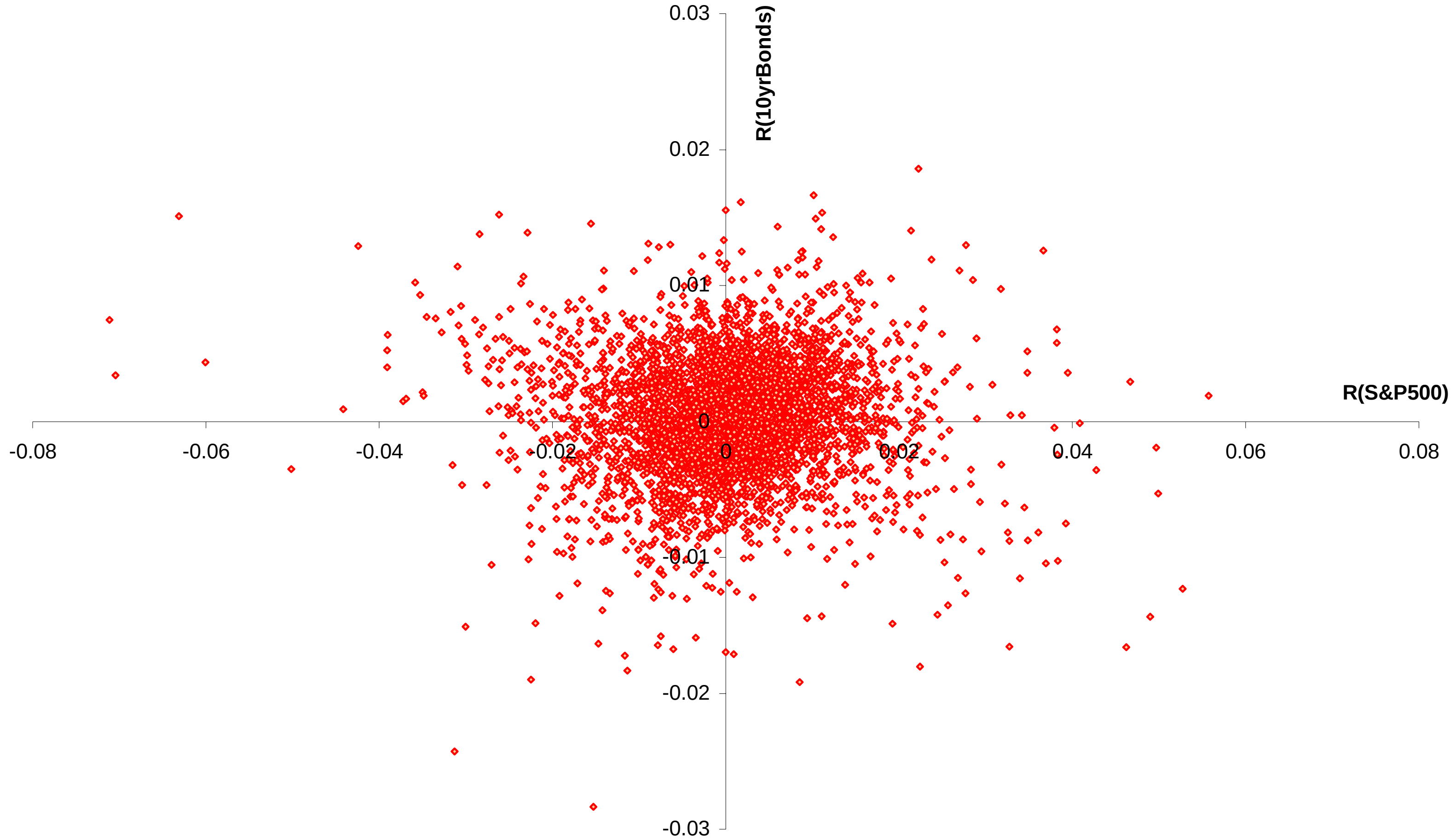
A US portfolio



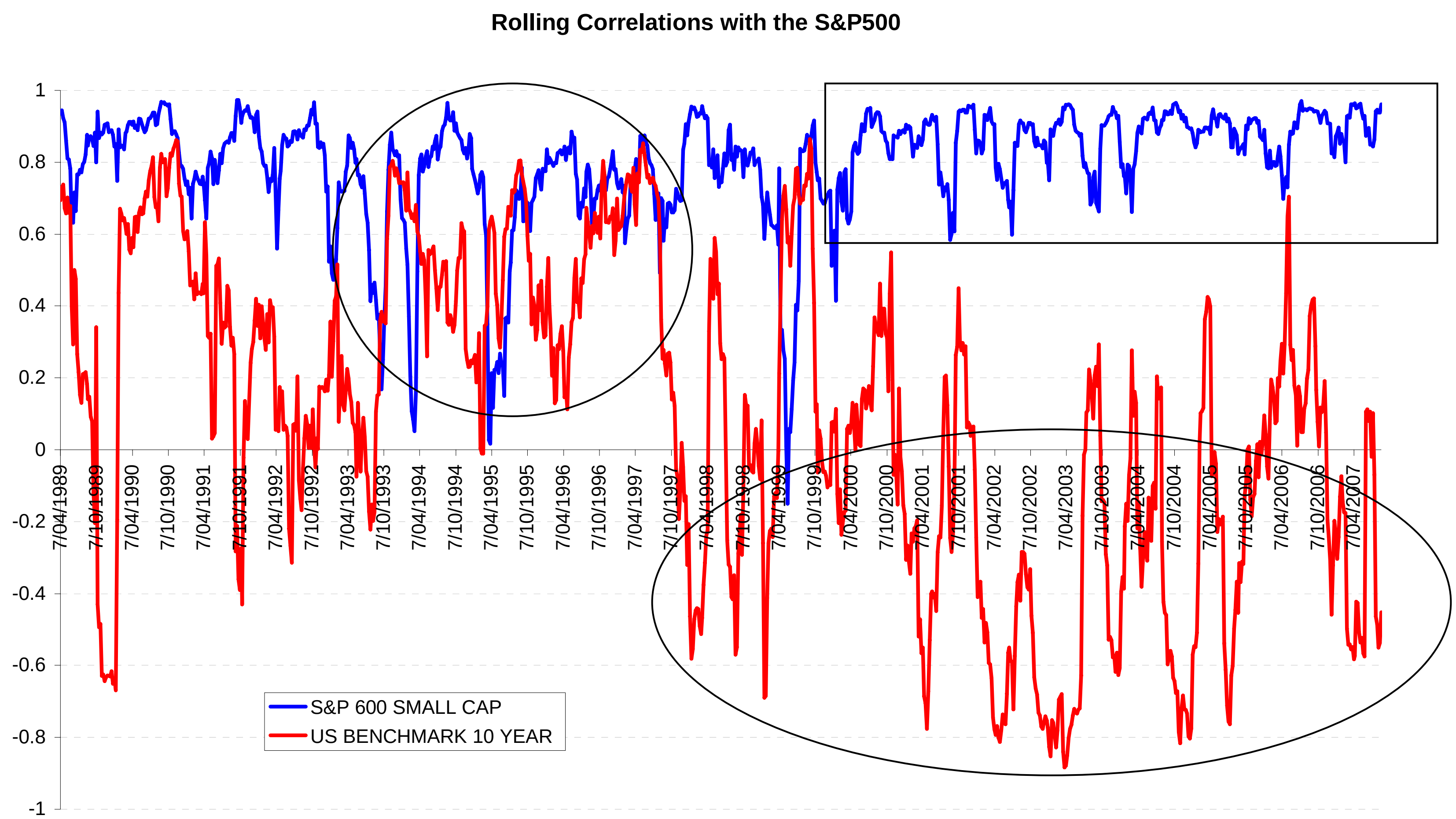
Elliptical bivariate distributions



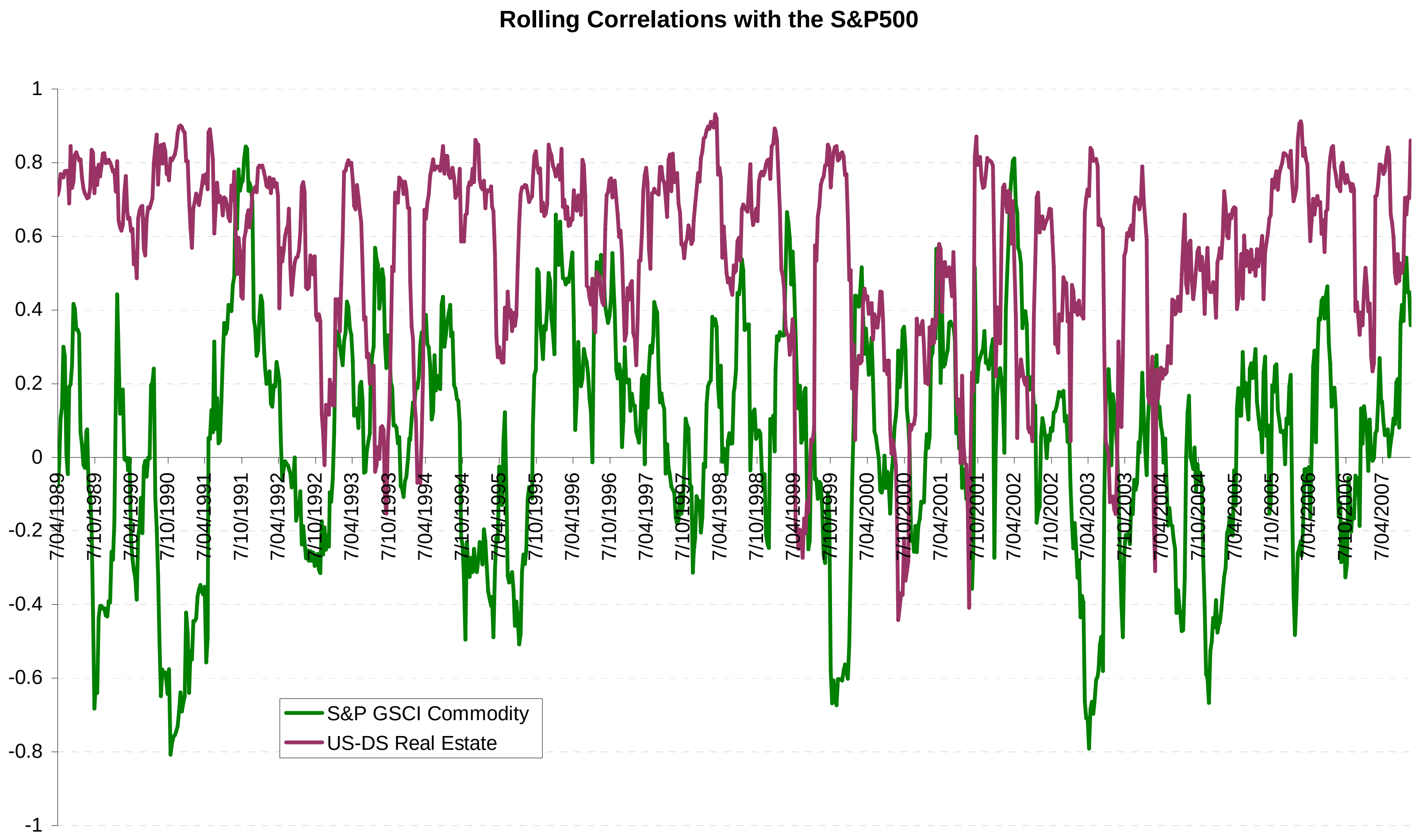
Non-elliptical bivariate distributions



Time-varying correlations



Time-varying correlations



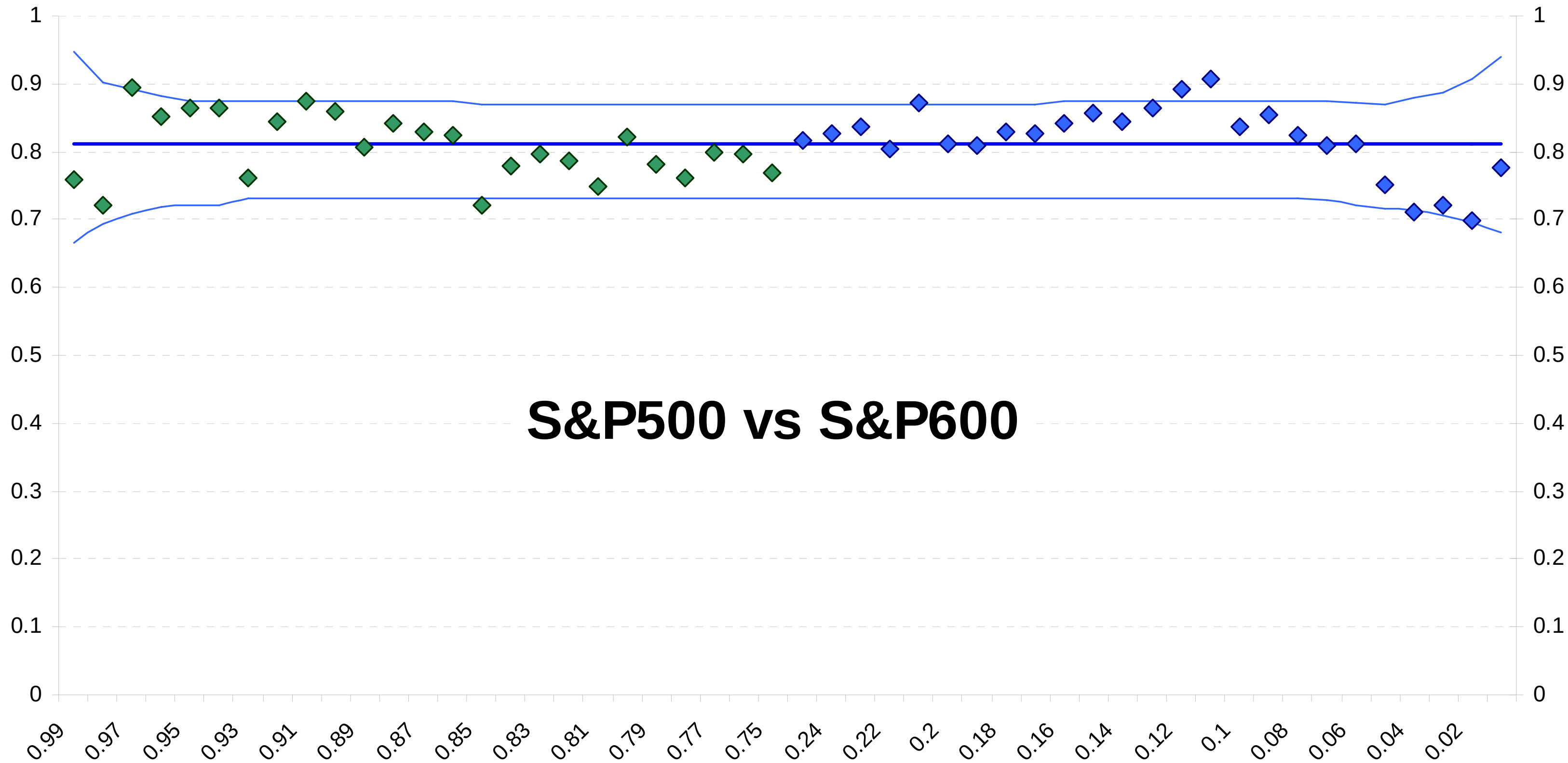
Portfolio parameter estimates

	S&P500	*S&P600*	*USCB*	*USGB10*	*GSCI*	*USREI*
S&P600	**0.81** 6.4					
USCB	**0.09** 5.4	**0.01** 6.2				
USGB10	**0.01** 5.4	**-0.05** 6.4	**0.94** 3.8			
GSCI	**-0.06** 6.9	**-0.01** 7.8	**-0.06** 7.2	**-0.05** 7.4		
USREI	**0.50** 5.9	**0.54** 6.2	**0.13** 5.6	**0.09** 5.7	**-0.07** 6.9	
	5.9	7.2	5.6	6.1	7.1	5.4

Unconditional correlations in bold; bivariate *t* degrees of freedom; and univariate *t* degrees of freedom

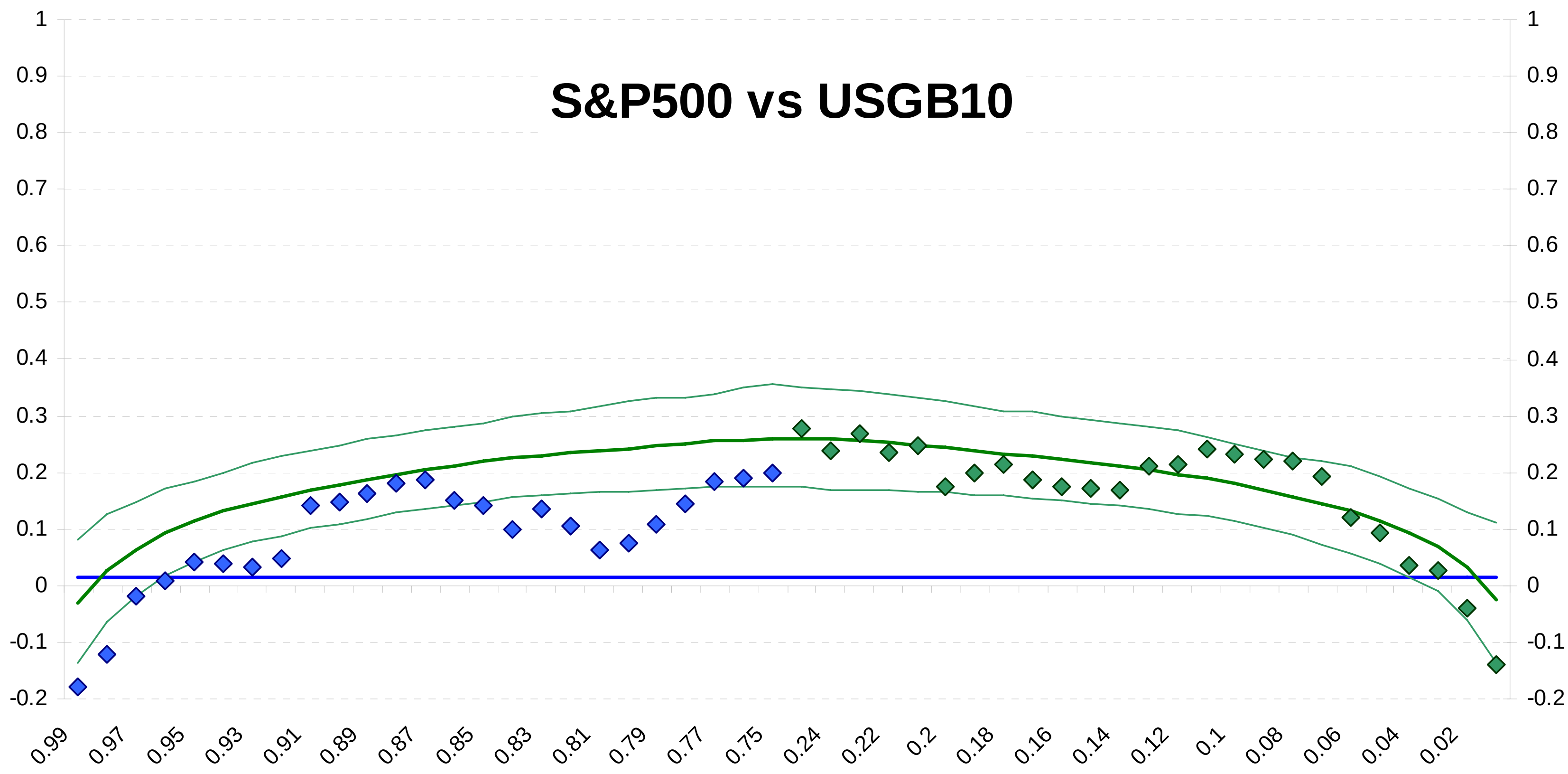
VaR-implied correlations

bivariate normal



VaR-implied correlations

bivariate *t*



Portfolio implications

Diversification benefits should erode quickly if the alleged increase in correlation occurs where diversification is most needed!

Particularly relevant for downside risk concerned investors.

Arzac and Bawa (1977, *JFE*); Leibowitz and Kogelman (1991, *JPM*)

lower partial moments risk measure (safety-first investors)

Harlow (1991, FAJ)

portfolio optimisation based on downside-risk constraints

Campbell, Huisman and Koedijk (2001, *JBF*)

Mean-VaR portfolio optimisation could use quantile-dependent correlations

Portfolio implementation

Under multivariate normality and constant correlation, *Mean-VaR* and *Mean-* σ optimal portfolios coincide.

If we find that *VaR*-implied correlations increase(decrease) in bivariate quantiles (q_c), then the Mean-*VaR* frontier will shift outwards (inwards) and the optimal portfolio will be VaR_c-dependent.

We can then compute the VaR_c-optimal efficient frontier and measure the 'sub-optimality' penalty when an investor ignore VaR_c-dependency.

We share with the dynamic asset allocation literature that our model does not (necessarily) reflect the representative agent, as it does not address market equilibrium.

Portfolio implementation

1. Compute the optimal *VaR*-portfolio given constant correlation under multivariate normality;
2. Given the optimal *VaR*-portfolio weights, estimate the *VaR*-implied correlations allowing for fat-tailed bivariate distributions;
3. Given the *VaR*-implied correlations, compute the new (updated) optimal VaR_c-portfolio(s) at different confidence levels 1-*c* (say 95%, 97.5%, 99%);
4. For each set of VaR_c-optimal portfolio weights, estimate the VaR_c-implied correlations;
5. Reiterate until convergence.

THE UNIVERSITY OF MELBOURNE

Conclusions

Impressions just might be deceiving!

Allowing for fatter-tailed distributions may absorb some of the apparent excesses in tail correlations.

This paper uses *VaR*-implied correlations and finds:

- Little evidence of asymmetric excess correlation

  …after conditioning on the ‘correct’ distribution.
- A better fit for Student-t bivariate correlations,

  …but, it is fairly marginal.

Work to do:

- Alternative models to capture switching regimes and clustering
- Alternative ways to characterize tail correlations
- Better understanding of the uses of conditional correlations