Jump Risk, Stock Returns, and Slope of Implied Volatility Smile*

Shu Yan†

Moore School of Business

University of South Carolina

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†Columbia, SC 29208, phone: (803) 777-4925, e-mail: syan@moore.sc.edu.

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Abstract

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JEL classification: G12

Key words: Jump risk; Stock returns; Options; Implied volatility smile; Slope

1 Introduction

It is well documented in the finance literature that distributions of stock returns are leptokurtic or "fat tailed". Excessive kurtosis of stock returns can be caused by jumps, that is, sudden but infrequent movements of large magnitude in stock prices. Modeling dynamics of jumps in stock prices dates back to Press (1967) and Merton (1976a). Subsequent studies such as Ball and Torous (1983), Jarrow and Rosenfeld (1984), and Jorion (1989) present convincing evidence for the presence of jumps in stock prices. Another strand of papers, following the approach of Cox and Ross (1976) and Merton (1976b), examine the effects of jumps in option pricing beyond the classical diffusion model of Black and Scholes (1973). Papers including Ball and Torous (1985), Naik and Lee (1990), Bakshi, Cao, and Chen (1997), Bates (2000), Duffie, Pan, and Singleton (2000), Anderson, Benzoni, and Lund (2002), Pan (2002), and Eraker, Johannes, and Polson (2003) find that incorporating jumps is essential in fitting observed option prices. Despite the theoretical importance and overwhelming empirical evidence for jumps, there is a lack of understanding on the relation between jump risk and cross-sectional expected stock returns. In this paper, we examine two open questions: (i) How is the expected return of a stock related to jump risk; and (ii) How do we measure jump risk?

To answer the first question, we adopt the stochastic discount factor (SDF) framework because of its simplicity and universality.[1] In the absence of arbitrage, there exists a positive SDF that prices all assets. (See, for example, Rubinstein (1976), Ross (1978), and Harrison and Kreps (1979).) Given the empirical evidence that stock prices contain systematic jumps, the SDF must contain jumps as well. We present a very general yet parsimonious continuous-time model where the SDF and stock prices follow correlated jump-diffusion processes. There are two sources of risk in the stock price dynamics: diffusive risk and jump risk, characterized by a Brownian motion and a Poisson process respectively. In this model, the expected excess stock return is dependent on both sources of risk. The diffusive component of the stock return is determined by the covariance between the Brownian motions driving the SDF and

[1] An excellent reference for the SDF approach in asset pricing is Cochrane (2005).

stock processes, a continuous-time analogue of the discrete-time β-representation. The jump component of the stock return is captured by: (i) the covariance between the Poisson processes in the SDF and stock; (ii) the covariance between jumps in the SDF and stock; and (iii) the average jump sizes of the SDF and stock.

Applying the jump-diffusion model to the data leads us to the second question. Among other things, we need to know/estimate distributions of jumps in the SDF and stock prices. There are a couple of major challenges. First, the SDF is not identified and thus its jump distribution is unknown. Naik and Lee (1990), for example, demonstrate that the market is incomplete when jumps are present in stock prices. And market incompleteness implies non-uniqueness of SDF. Fortunately, existing asset pricing models provide us some guidance on the sign of average SDF jump size. In the classic CAPM, for example, the SDF is inversely related to the market portfolio, and consequently jumps in the SDF are negative of the jumps in the market portfolio. Then the strong evidence of negative average jumps in the market portfolio documented by Bakshi, Cao, and Chen (1997), Bates (2000), Pan (2002), Eraker, Johannes, and Polson (2003) among others implies positive average SDF jump size in the CAPM. As a second example, the SDF in the consumption-based CAPM is the intertemporal marginal rate of substitution, and the average SDF jump size is positive if the average jump size in the aggregate consumption is negative. This is consistent with the assumption of Rietz (1988) in his explanation of the equity premium puzzle of Mehra and Prescott (1985), and is supported by the empirical evidence documented in Barro (2006). At least according to these asset pricing models, the average SDF jump size is positive although other jump parameters are undetermined unless a particular SDF is specified and estimated. One important implication of the positive average jump size of the SDF is that, for reasonable model parameters, the excess stock return is decreasing in the average jump size of the stock.

The second challenge in empirically implementing the jump-diffusion model is that distributions of jumps in stock prices are difficult to estimate because jumps are rare events and long samples of stock prices are often unavailable. Moreover, high probability jumps may fail to realize in sample due to the Peso problem.[2] Making the matter worse, the jump risk of a stock may be time-varying as the joint distribution of jumps in the stock and SDF can change over time. We solve this problem by using information from the option market. There are two advantages in using option data. First, the option and stock markets trade simultaneously and their information contents are synchronized. Second but more importantly, options are forward looking contracts and thus are informative about future expected returns. This mitigates the Peso problem and reduces the bias caused by in-sample fitting of time-varying jump distributions.

In this paper, the option variable we use is the *slope* of implied volatility smile,[3] defined to be the difference between fitted implied volatilities of one-month-to-expiration put and call options with option delta equal to -0.5 and 0.5 respectively. We show theoretically that slope indeed measures the local steepness of the smile for near-the-money near-expiration options. Furthermore, we prove that slope is approximately proportional to the average jump size of the underlying stock price. Empirically, we find evidence that slope predicts future stock jump sizes. For example, we find that high slope leads to high return skewness, consistent with the presence of large positive jumps. We also use the method of Jiang and Yao (2007) to estimate realized stock jump sizes. Our predictive regression results indicate that slope is positively related to future realized jump sizes. We combine these results with the assertion that expected stock return is decreasing with average stock jump size to obtain our main testable hypothesis on cross-sectional stock returns: If stock portfolios are formed by ranking on the slope of implied volatility smile, then the future returns of high slope portfolios are lower than those of low slope portfolios.

Option data have been used extensively for estimating dynamics of stock prices. But most earlier papers focus on index options and use advanced estimation methods. Our approach is easy

[2] Significant progress has been made in estimating jumps in asset prices. Recent papers include Bates (1996), Bakshi, Cao, and Chen (1997), Anderson, Benzoni, and Lund (2002), Pan (2002), Carr and Wu (2003), Chernov, Gallant, Ghysels, and Tauchen (2003), Eraker, Johannes, and Polson (2003), Ait-Sahalia (2004), and Jiang and Yao (2007) among others.

[3] It is well-known that implied volatilities of equity options exhibit a "smile" shape. See, for example, MacBeth and Merville (1979) and Rubinstein (1985).

to implement and does not require long time-series samples. Furthermore, our approach allows flexible specification of the stock price process that can incorporate, for example, stochastic volatility. In contrast, previous studies often use certain parametric models. We should point out that our analysis is under the joint assumption that the asset pricing model is correct and (short-term near-the-money) options are priced accordingly. If either the model is misspecified or there are systematic option pricing errors, our explanation of jump risk in terms of slope of implied volatility smile may not be valid.[4]

We test the proposed hypothesis using the option data on 4,048 stocks from January, 1996 to June, 2005. At the end of each month, five equally weighted quintile portfolios are formed by sorting stocks on slope. Indeed, the average portfolio returns in the subsequent month exhibit a monotonic decreasing pattern in slope, confirming the prediction of our hypothesis. The difference between the average monthly returns of the lowest quintile and highest quintile portfolios is 1.8%. The pattern cannot be explained by the four-factor model of Carhart (1997) that includes the three factors of Fama and French (1993), and the momentum factor based on Jegadeesh and Titman (1993). The pattern is also robust after controlling for a number of stock characteristics including:[5] market β, past return, past idiosyncratic return, size, book-to-market, leverage, implied volatility, idiosyncratic implied volatility, historic idiosyncratic volatility, historic skewness, co-skewness, proportion of systematic risk, option trading volume, stock trading volume, and stock turn-over. The pattern suggests a profitable yet risky zero-cost trading strategy by long the lowest quintile portfolio and short the highest quintile portfolio. For our sample period, the annualized profit of this strategy is 22.2%, and it remains as high as 13.4% even after accounting for a 1% round-trip transaction cost. The profitability of the long-short strategy is persistent up to six months although most of the profit comes from the first month. Our findings are not driven by the choice of data set and definition of slope.

[4] Coval and Shumway (2001), Driessen and Maenhout (2007), and Goyal and Saretto (2007) document evidence of some economically profitable and statistically significant option trading strategies, which may indicate mispricing. The evidence, however, can be consistent with our model if the profits generated by these option strategies compensate investors for bearing the jump risk.

[5] The variables chosen here are motivated by a long list of studies like Banz (1981), Basu (1983), Rosenberg, Reid, and Lanstein (1985), Fama and French (1992), Jegadeesh and Titman (1993), Lakonishok, Shleifer, and Vishny (1994), Harvey and Siddique (2000), and Ang, Hodrick, Xing, and Zhang (2006) among others.

Our slope variable differs from the various measures for steepness of implied volatility smile in the literature in two ways. First, the slope in this paper is a point estimate for the local steepness while previous definitions generally measure steepness in a global fashion. The put and call options used to define our slope are very close to at-the-money. But other definitions of slope use options with very different strike prices. Second, our asset pricing model relates slope directly to the jump risk of the underlying stock while other papers use different interpretations. Among recent papers, Toft and Prucyk (1997) relate slope to firm leverage; Dennis and Mayhew (2002) and Bakshi, Kapadia, and Madan (2003) demonstrate the connection between slope and skewness of the risk neutral density; Cremers, Driessen, Maenhout, and Weinbaum (2004) document relation between slope and credit spread; Bollen and Whaley (2004) show slope to be affected by the net buying pressure from public order flow; Duan and Wei (2007) find that slope is dependent on the systematic risk proportion in the total risk.

The papers closest to ours include Pan and Poteshman (2006), Zhang, Zhao, and Xing (2008), Conrad, Dittmar, and Ghysels (2008), and Rehman and Vilkov (2009). Pan and Poteshman (2006) find that stocks with low put-call ratios outperform stocks with high put-call ratios by more than 40 basis points on the next day and more than 1% over the next week. However, their put-call ratio is constructed from nonpublic information. In contrast, slope is publicly observable. Zhang, Zhao, and Xing (2008) use their *skew* measure to show that low skew stocks outperform high skew stocks by about 18% per year. Based on the informed trading model of Easley, O'Hara, and Srinivas (1998), they argue that their findings reflect informed investors' demand of out-of-the-money puts in anticipating bad news about future stock prices. In contrast, our model assumes efficient stock market and option market, and slope measures the jump risk. In fact, a high slope in our model predicts good news (positive jumps). We present empirical evidence that the *skew* of Zhang, Zhao, and Xing (2008) cannot explain the stock return pattern in slope while slope has explanatory power on *skew*. Conrad, Dittmar, and Ghysels (2008) use the model of Bakshi, Kapadia, and Madan (2003) to show that stocks with high (low) option implied skewness have low (high) future returns. Their findings are consistent with ours as high slope stocks are more likely to have large positive jumps and consequently high skewness. On the contrary, Rehman and Vilkov (2009) use a similar method but find high skewed stocks out-perform low skewed stocks. Both Conrad, Dittmar, and Ghysels (2008) and Rehman and Vilkov (2009) interpret their findings as investors' preference/sentiment to higher moments of stock returns whereas we use a no-arbitrage asset pricing model that incorporates jump risk.

Our paper is also related to recent studies such as Ofek, Richardson, and Whitelaw (2004) and Cremers and Weinbaum (2008) that find deviations from put-call parity can predict future stock returns. For example, Cremers and Weinbaum (2008) show that stocks with relatively expensive calls outperform stocks with relatively expensive puts by 50 basis points per week, which is consistent with our findings if we loosely interprete lower value of slope as relative expensive call option and higher value of slope as relatively expensive put option. However, there are some important differences. First, the put and call options used in our slope variable do not have the same strike price. So technically a non-zero slope cannot be interpreted as deviation from put-call parity. Second, previous studies consider all traded put-call option pairs while we use only one put-call option pair to define slope. Third, these papers attribute the predictability of stock returns by deviations from put-call parity to limits of arbitrage and mispricing in the stock market. In contrast, we assume that investors are rational and slope just reflects jump risk.

The paper proceeds as follows. In section 2, we present the jump-diffusion model and all the theoretical results. Section 3 contains the empirical analysis of slope and cross-sectional stock returns. In section 4, we conduct robustness checks. Section 5 concludes. Technical results are provided in the Appendix.

2 Jump-Diffusions and Asset Pricing

In this paper, we take the continuous-time approach as jumps are natural to formulate under this framework. A stochastic discount factor (SDF), $M(t)$, is a positive stochastic process so that MS_i is a martingale for any stock price process $S_i(t)$. This condition is often represented by the following identity:

$$\mathrm{E}_t\left[d(MS_i)\right] = 0, \tag{1}$$

where $\mathrm{E}_t[.]$ is expectation conditional on information available at time t. Most existing asset pricing models can be unified under the stochastic discount factor framework. In the consumption-based CAPM of Breeden (1979), for example, M is equal to the intertemporal marginal rate of substitution of the representative investor.

2.1 Stochastic Discount Factor and Stock Returns

Let $M(t)$ be a SDF following the jump-diffusion process:[6]

$$\frac{dM}{M} = \left(-r_f - \lambda_M \mu_{J_M}\right) dt + \sigma_M dW_M + J_M dN_M, \tag{2}$$

where W_M is a standard Brownian motion and N_M is a Poisson process with intensity $\lambda_M (\geq 0)$, that is, $\text{Prob}(dN_M = 1) = \lambda_M dt$. J_M is the jump size with a displaced lognormal distribution independent over time:

$$\ln(1 + J_M) \sim \mathcal{N}\left(\ln(1 + \mu_{J_M}) - \frac{1}{2}\sigma^2_{J_M}, \sigma^2_{J_M}\right). \tag{3}$$

The lognormal specification of J_M ensures positivity of M, which guarantees no arbitrage. W_M, N_M, and J_M are assumed to be independent of each other. r_f is the risk-free interest rate. The term $\lambda_M \mu_{J_M}$ adjusts the drift for the average jump size. σ_M is the instantaneous

[6] This type of models are first introduced by Merton (1976a). We use one-dimensional Brownian motion and Poisson process for simplicity. The model can be extended to incorporate multi-dimensional Brownian motions and Poisson processes.

diffusive standard deviation. Similarly, we model the price of the i-th stock as a jump-diffusion process:

$$\frac{dS_i}{S_i} = (\mu_i - \lambda_i \mu_{J_i})\, dt + \sigma_i dW_i + J_i dN_i, \tag{4}$$

where W_i is a standard Brownian motion and N_i is a Poisson process with intensity λ_i. J_i has a displaced lognormal distribution independent over time:

$$\ln(1 + J_i) \sim \mathcal{N}\left(\ln(1 + \mu_{J_i}) - \frac{1}{2}\sigma_{J_i}^2, \sigma_{J_i}^2\right). \tag{5}$$

Again W_i, N_i, and J_i are assumed to be independent of each other, but they are related to the corresponding components of the SDF. We assume that W_M and W_i, N_M and N_i, and J_M and J_i are pairwise correlated with correlation coefficients $\mathrm{Corr}\,(dW_M, dW_i) = \rho_i$, $\mathrm{Corr}\,(N_M, N_i) = \eta_i$, and $\mathrm{Corr}\,(\ln(1 + J_M), \ln(1 + J_i)) = \psi_i$ respectively. Notice that η_i has to be at least zero while ρ_i and ψ_i can be negative. We show the following proposition in the Appendix.

Proposition 1: *Given the dynamics of the SDF and stock price in* (2)-(5)*, the excess stock return can be expressed as:*

$$\mu_i - r_f = -\rho_i \sigma_M \sigma_i - \eta_i \sqrt{\lambda_M \lambda_i}\left[(1 + \mu_{J_M})(1 + \mu_{J_i}) e^{\psi_i \sigma_{J_M} \sigma_{J_i}} - \mu_{J_M} - \mu_{J_i} - 1\right]. \tag{6}$$

Moreover, the excess stock return is (i) decreasing in ρ_i and ψ_i; (ii) decreasing (increasing) in η_i if $\Theta_i \equiv (1 + \mu_{J_M})(1 + \mu_{J_i}) e^{\psi_i \sigma_{J_M} \sigma_{J_i}} - \mu_{J_M} - \mu_{J_i} - 1 > 0$ (< 0); and (iii) decreasing (increasing) in μ_{J_i} if $\Phi_i \equiv (1 + \mu_{J_M}) e^{\psi_i \sigma_{J_M} \sigma_{J_i}} - 1 > 0$ (< 0).

Some remarks are warranted. First, in the absence of jumps, only the first term on the right hand side of (6) remains. This is the well-known continuous-time analogue of the discrete-time β-representation of expected returns. In particular, the return of a stock that is negatively correlated with the SDF ($\rho_i < 0$) is higher than the risk-free rate. Second, when jumps are present but non-systematic ($\eta_i = 0$), (6) is the same as that in the case of no jumps. This is exactly what Merton (1976a) argues that idiosyncratic (diversifiable) jumps do not affect expected stock returns. However, in the presence of systematic jump risk ($\eta_i > 0$), the excess stock return depends on the jump distributions. Proposition 1 says that stocks whose systematic jumps are more negatively correlated with jumps of the SDF ($\psi_i < 0$) earn higher returns *ceteris paribus*. However, the relationship between η_i and excess stock return and the relationship between μ_{J_i} and excess stock return depend on the signs of quantities Θ_i and Φ_i, respectively. For the rest of the paper, we focus on the relationship between μ_{J_i} and excess stock return as we are able to estimate μ_{J_i} from the data.

To understand to the effect of μ_{J_i} on stock returns, we first consider the special case of $\psi_i = 0$, that is, systematic stock jumps are uncorrelated with jumps of the SDF. Now, the key quantity Φ_i simplifies to μ_{J_M}. Even under such strong assumption, we still need to know the sign of μ_{J_M}, the average jump size of M, which is not clearly specified in our model. The main problem is that SDF is not unique because the market is incomplete due to the presence of jumps. Fortunately, some well-known asset pricing models hint that $\mu_{J_M} > 0$. For example, in the CAPM theory, $M = a - bR_M$, where R_M is the return of the market portfolio, and a and b are positive constants.[7] Together with the empirical evidence that the average jump size of the market portfolio is negative, this implies $\mu_{J_M} > 0$. As another example, the SDF in the consumption-based CAPM of Breeden (1979) is proportional to the intertemporal marginal rate of substitution. For a representative investor with a time-separable power utility function, jumps in the SDF are negatively related to jumps in the consumption growth.[8] Therefore, $\mu_{J_M} > 0$ holds if the average jump in consumption is negative. This is exactly what Rietz (1988) assumes in his explanation of the equity premium puzzle of Mehra and Prescott (1985). Recently Barro (2006) documents evidence supporting this assumption. In summary, it is reasonable to assume $\mu_{J_M} > 0$, and then (6) indicates that excess stock returns

[7]It can be shown that $a = \frac{1}{1+r_f}$ and $b = \frac{\bar{R}_M - r_f}{(1+r_f)\sigma_M^2}$, where $\bar{R}_M$ is the average market return and σ_M is the standard deviation of market return. To guarantee $b > 0$, we need $\bar{R}_M > r_f$. Note that the classical CAPM of Sharpe (1964) and Lintner (1965) is a discrete time static model. To be consistent with the continuous time framework, we can use the intertemporal CAPM of Merton (1973). In the simplest form of ICAPM where wealth is the only state variable, M is proportional to marginal utility of wealth and jumps in M are negatively proportional to jumps in total wealth, which is generally proxied by the market index. (See, for example, Cochrane (2005).)

[8]Details can be found in, for example, Cochrane (2005).

are monotonically decreasing in average stock jump size ($\Phi_i > 0$) when jumps in stocks are uncorrelated with jumps in the SDF ($\psi_i = 0$).

In general, ψ_i for a stock can be negative or positive. The monotonicity of excess stock return in μ_{J_M} is determined by the values of μ_{J_M}, ψ_i, σ_{J_M}, and σ_{J_i}. As a benchmark, we consider the case where CAPM holds and M is proportional to the inverse of the market portfolio. We let $\mu_{J_M} = 10\%$ and $\sigma_{J_M} = 15\%$ as these values are consistent with those in the literature.[9] As one worst scenario against $\Phi_i > 0$, we let $\psi_i = -1$, and further let $\sigma_{J_i} = 40\%$, which is very generous as it is more than two-thirds of the average standard deviation of realized stock returns in our sample. Even for these extreme values of ψ_i and σ_{J_i}, we have $\Phi_i = 0.036 > 0$. It seems that $\Phi_i > 0$ holds as long as μ_{J_M} and σ_{J_M} are of similar magnitude and the product $\psi_i \sigma_{J_i}$ is not too negative, which can be due to either small ψ_i or reasonable size of σ_{J_i}. Of course, it is possible that $\Phi_i < 0$ for some stocks. But these stocks should be outnumbered by stocks with $\Phi_i > 0$ in well-diversified portfolios. Therefore, we conclude that the excess stock return is generally monotonically decreasing in μ_{J_i}.

2.2 Jumps and Slope of Implied Volatility

Equations (6) demonstrates how the expected excess stock return is dependent on the systematic jump risk. To apply these asset pricing equations in practice, we need to know, in addition to parameters related to the SDF process, parameters for the stock price process such as ρ_i, σ_i, η_i, λ_i, ψ_i, μ_{J_i}, and σ_{J_i}. As argued in Merton (1980), the parameters such as ρ_i and σ_i related to the diffusive risk can be accurately estimated by quadratic (co)variation of realized stock returns. However, the jump parameters such as μ_{J_i} and σ_{J_i} are difficult to estimate because jumps are rare events and may fail to materialize in sample. Accurate estimation requires very long time series of stock returns, which are not often available. Moreover, these

[9] For the same sample period of 1/1995 - 6/2005, the estimates of the average market jump size and standard deviation for the most general SV-SJ model in Santa-Clara and Yan (2008) are -9.8% and 16% respectively. There are other estimates in the literature for different sample periods. For example, the estimates of the SVJ model in Bakshi, Cao, and Chen (1997) are -5% and 7% respectively while the estimates of the SVIJ model in Eraker, Johannes, and Polson (2003) are about -3% and 4% respectively. But we still have $\Phi_i > 0$ using these parameter values.

parameters may be time-varying and historic estimates can be biased. In this paper, we propose a rather simple method to infer the forward-looking average stock jump size (μ_{J_i}) that uses information from the option market, which is readily available. The intuition arises from the ground-breaking work of Merton (1976a) where he demonstrates how jumps affect option pricing. Conversely, from the observed implied volatility smile, we can obtain information about the underlying jump distribution of stock price process.

Let $S_i(t)$ be the price of the i-th stock, which follows the jump-diffusion process defined by equation (4). Denote the stock dividend yield by q_i, assumed to be constant. We consider a European call option on the stock with strike price K and maturity T. Let $\sigma_i^{\text{imp}}(K,T)$ denote the Black-Scholes option implied volatility. We define *log moneyness* to be $X \equiv \ln\left(Ke^{-(r_f-q_i)T}/S_i(0)\right)$, which is more convenient to work with than K.[10] Without ambiguity, we write the implied volatility as $\sigma_i^{\text{imp}}(X,T)$.

It is well-known in the literature that option implied volatilities exhibit a "smile" shape. We show, in the following proposition, that the local steepness of the smile for at-the-money near-expiration options is related to the average stock jump size (and jump intensity).

Proposition 2: *For T small, the Black-Scholes implied volatility of the at-the-money European call option satisfies*:[11]

$$\left.\sigma_i^{\text{imp}}(X,T)\right|_{X=0} = \sigma_i + O(T), \tag{7}$$

$$\left.\frac{\partial \sigma_i^{\text{imp}}(X,T)}{\partial X}\right|_{X=0} = \frac{\lambda_i \mu_{J_i}}{\sigma_i} + O(T), \tag{8}$$

where $O(T)$ 'means in the same order as T'.

Equation (7) says that the implied volatility of an at-the-money European call option approaches the instantaneous diffusive volatility of stock returns as time-to-maturity approaches zero. Ledoit, Santa-Clara, and Yan (2003) prove the same result for diffusion processes. Propo-

[10] In the literature, moneyness is often defined as K/S. Our log-transformed definition takes into account of time value and leads to cleaner formula.

[11] Technically, the parameters such as λ_i and μ_{J_i} should be specified under the risk-neutral probability measure. See the Appendix for further discussions on this issue. The proposition also holds for put options.

sition 2 shows that equation (7) holds even in the presence of jumps. In other words, jumps do not affect the level of short-term at-the-money implied volatility. Interestingly we see from equation (8) that jumps affect the local steepness of implied volatility smile near-the-money. For short maturity, the slope is proportional to the average stock jump size. Since both σ_i and λ_i are positive, the sign of the slope is identical to the sign of μ_{J_i}. So the slope is positive (negative) if the average jump size is positive (negative).[12]

One concern about Proposition 2 is how big the approximation errors in equations (7) and (8) are. In the Appendix, our simulations show that the errors in implied volatility are small while the errors in slope can be significant for long maturities. This is not surprising because slope is a derivative of implied volatility and its error should be of higher order than that in the level of implied volatility. Nonetheless, our simulation results indicate that slope is an increasing function of the average jump size μ_{J_i} even in the presence of approximation errors.

To implement the results of Proposition 2, we fix time-to-maturity to be small and consider implied volatility $\sigma^{\text{imp}}_{i,\text{put}}$ ($\sigma^{\text{imp}}_{i,\text{call}}$) of the put (call) option on the i-th stock with $\Delta = -0.5$ (0.5).[13] Define implied volatility (v_i) and slope of implied volatility smile (s_i) as following:[14]

$$v_i \equiv 0.5(\sigma^{\text{imp}}_{i,\text{put}} + \sigma^{\text{imp}}_{i,\text{call}}), \tag{9}$$

$$s_i \equiv \sigma^{\text{imp}}_{i,\text{put}} - \sigma^{\text{imp}}_{i,\text{call}}. \tag{10}$$

In the Appendix, we prove the next proposition.

[12] It is important to note that the proposition still holds even when the diffusive volatility and average jump size are time-varying. The proof in the Appendix can be extended to accommodate such more general models. Indeed, the findings of Bakshi, Cao, and Chen (1997), Bates (2000), and Santa-Clara and Yan (2008) among others strongly support this kind of specification. In the Appendix, we use the Monte-Carlo simulation method to show that Proposition 2 holds even for a stochastic volatility model.

[13] As we see later, the implied volatility data on individual stocks with fixed maturity and option delta are readily available. Technically, the put and call options used are not exactly at-the-money but only very close to being at-the-money.

[14] One practical problem is that individual equity options are American style and their implied volatilities are not obtained by inverting the Black-Scholes formula. Strictly speaking, our theoretical results cannot be directly applied to the data available. Nonetheless, because the options that we use in the empirical analysis are short-term and near-the-money contracts, their prices are close to the prices of similar European options as early exercise value is low. For example, Bakshi, Kapadia, and Madan (2003) examine a sample of 30 largest stocks in the S&P 100 index and find the difference between Black-Scholes and American option implied volatilities is small enough to be ignored. Therefore, the slope constructed from these options should be approximately equal to that from similar European options.

Proposition 3: *Defined as in* (9) *and* (10), v_i *is approximately equal to the diffusive volatility* σ_i *and* s_i *is approximately proportional to the product of jump intensity and average stock jump size. That is,*

$$v_i \approx \sigma_i, \tag{11}$$

$$s_i \approx L_i \lambda_i \mu_{J_i}, \tag{12}$$

where $L_i > 0$ *is a constant.*

The most interesting result of Proposition 3 is equation (12), which says that the slope of implied volatility smile is increasing in terms of the average stock jump size. Comparing it with (8), we see that s_i is indeed proportional to the local steepness of the implied volatility smile.[15] Combining this with the discussions following Proposition 1 in Section 2.1, we can argue that the return of a well-diversified stock portfolio is decreasing in the average slope of implied volatility smile. This leads to our main hypothesis to be tested empirically.

Hypothesis: If we form portfolios of stocks by ranking on slope of implied volatility smile, then the returns of high slope portfolios are larger than the returns of low slope portfolios.

3 Empirical Analysis

In this section, we first discuss the data used in the paper. Then we present evidence that slope of implied volatility smile predicts future stock jump size. Next, we test the proposed hypothesis in the last section by examining the performance of portfolios formed on slope. Finally, we examine the portfolio performance by controlling for a number of stock characteristics.

[15] To follow (8) exactly, we should use s_i/v_i as the definition of slope. We choose the current version for simplicity and its direct relation to the average jump size. Later we conduct robustness test in our empirical analysis and find qualitatively and quantitatively similar results using this alternative definition of slope.

3.1 Data

We use the stock option data from OptionMetrics for the period of January, 1996 – June, 2005. As individual equity options are American style, OptionMetrics employs an algorithm based on the Cox-Ross-Rubinstein binomial tree model (Cox, Ross, and Rubinstein 1979) to calculate implied volatility of option contracts. The implied volatility surface is then constructed from the observed implied volatilities with a kernel smoothing technique.[16] OptionMetrics reports the fitted implied volatilities (of both puts and calls) on a grid of fixed maturity and option delta.[17] For each stock, we collect the end-of-month (last trading day) fitted implied volatilities of put and call options with one month ($T = 1/12$) to expiration and $\Delta = -0.5$ and 0.5 respectively. We use equations (11) and (12) to calculate v and s for the stock.[18]

On the last trading day of a month, we match the option data with stock and accounting data obtained from the CRSP and COMPUSTAT. We exclude stocks that do not have at least two previous years of return data to estimate market beta. We use the market capitalization, book-to-market ratio, and leverage of each stock observed two quarters ago to define the variables ME, BM, and LV, respectively.[19] In this paper, we consider three measures of liquidity: OV is the total trading volume of all puts and calls on the stock during the month; SV is the total trading volume of the stock during the month; and TO is the turn-over rate of the stock during the month. The monthly data on the Fama-French factors ($R_M - R_f$, SMB, and HML) and the momentum factor (MOM) are obtained from Kenneth French's web site. We conduct empirical analysis at monthly frequency for two reasons. First, this is the frequency used by most studies on cross-sectional stock returns. For example, the Fama-French factors are available monthly. Second, our analysis at monthly frequency has the benefit of homogeneity.

[16] See OptionMetrics' data manual for a detailed explanation.

[17] OptionMetrics only includes a standard option is if there exists enough option price data on that date to accurately interpolate the required values.

[18] Notice that v and s are constructed from the fitted implied volatility surface. Traded put (call) options with exact one month to expiration and/or $\Delta = -0.5(0.5)$ do not generally exist. Early studies often use the implied volatility values of traded options with certain moneyness to define v and s. The main advantage of our method is that it uses the full option price data to estimate the implied volatility surface and obtain homogeneous (in terms of moneyness and maturity) estimates of v and s.

[19] We define the leverage to be the ratio of (book debt)/(market equity). Our results do not change if we use (book debt)/(book debt + market equity) to define leverage.

That is, the options used for estimating implied volatility surface in different months have similar maturities.

Table 1 reports the summary statistics of the data. There are 4,048 stocks in our sample with an average time-series length of 47 months. The mean market capitalization of the firms is over $3 billions and the mean book-to-market ratio is a little higher than 1. For our sample, the average firm leverage ratio is slightly higher than 2, the average monthly stock return is 1%, and the average β is more than 1.3. The stock returns are fatted tailed and positively skewed on average. The average stock implied volatility is 56.7%, more than twice of the average implied volatility of the S&P 500 index options (about 20%) for the same period.[20] The slope of implied volatility smile varies a lot across stocks (high standard deviation of s across stocks) and over time (high standard deviation of Δs) although it is positive on average. At the level, r, v, and s are not much correlated. The negative correlation between r and Δv confirms the well documented leverage effect suggested by Black (1976) and Christie (1982). In contrast, Δs is not significantly correlated with either r or Δv.

For each month, we rank stocks in ascending order according to s. Five quintile portfolios are formed by equally weighing the stocks within each quintile. On average, there are 402 stocks in each portfolio during our sample period. As a robustness check, we also form ten decile portfolios. When another variable such as market β is used to rank stocks in addition to slope, we follow the two-pass sort approach of Fama and French (1992) to form portfolios by ranking stocks on β first and then on s. That is, we form 5×5 quintile portfolios by first allocating stocks into five quintile portfolios using their pre-ranking βs and then dividing stocks within each β quintile equally into five portfolios using their rankings on s.

3.2 Slope Predicting Jumps

According to the theoretical results of the previous section, the slope of implied volatility smile should predict expected average jump size in stock returns. Particularly, higher slope stocks

[20] OptionMetrics also provides implied volatility data for stock index options. Implied volatilities on S&P 500 are obtained by inverting the Black-Scholes formula because these options are European.

should experience higher average-size jumps in the future. To test this implication empirically, we face two challenges. First, identifying realized jumps often requires long time series of stock returns. But even in long samples, jumps may not materialize as they are rare. The second challenge, which is closely related to the first one, is that jump distributions vary over time. This can be seen from the large time variation of slope, making identification of time-varying jumps even more difficult. In this paper, we use two different methods to test the relationship between slope and realized jumps.

In the first test, we use the well-known fact that jump and skewness are positively related as positive (negative) jumps induce positive (negative) skewness. If our jump-diffusion model is valid, then equation (12) implies that high slope should predict high future skewness. To take into account of time variation in slope, we compute the stock return skewness in a new way as following.

To ensure enough observations in computing sample moments, we only consider the 585 stocks that have implied volatility and slope data for the whole period. For the i-th stock, let $\{r_t^i\}_{t=1}^T$ denote its monthly return series. Define a *ranking* series $\{I_t^i\}$ so that $I_t^i = n$ if the slope of the stock in month $t-1$ is ranked in the n-th quintile where $n \in (1, ..., 5)$. Fixing a number $n \in (1, ..., 5)$, we collect observations in $\{r_t^i\}_{t=1}^T$ with slope ranking equal to n, that is, $\{r_{t_j}^i : I_{t_j} = n\}$. We then calculate the skewness of the sub-series $\{r_{t_j}^i : I_{t_j} = n\}$. For accurate estimation, we only consider sub-series with at least ten observations. So we have (at-most) five skewnesses for each stock corresponding to five slope rankings. Note that these are future skewness measures. According to our model, we expect the skewness of the sub-series $\{r_{t_j} : I_{t_j} = n\}$ be increasing in n.

Table 2 report the summary statistics of the five skewnesses across different stocks. As predicted by our theory, the average skewness increases from 0.075 for the lowest quintile to 0.327 for the highest quintile. That is, when the slope of a stock is in the top quintile, its skewness is more than four times higher than when its slope is in the bottom quintile. A direct t-test shows these two sample means are statistically different. Note, however, that the medians of five quintiles are very close. This implies there are more low skewnesses in quintile 1 while there are more high skewnesses in quintile 5. The evidence suggests that a stock is indeed more likely to have positive jumps when slope is high.

Our second test relies on the jump-identification methodology of Jiang and Yao (2007), which is based on the work of Jiang and Oomen (2005). We follow their procedure in identifying the presence of jumps and further estimating realized jump sizes. For a 12-month period ended in month t ($t-11, t-10, ..., t$), we use daily returns of a stock to construct the jump test statistic in Jiang and Yao (2007), which asymptotically follows a standard normal distribution. If the null of no jumps is rejected at the 5% critical level, we follow their approach to estimate the annual jump size during that year, and call it JR_t. If the null is not rejected at the 5% critical level, the annual jump size in that year is a missing observation.[21] We repeat these steps for the next 12-month period ($t-10, ..., t+1$), and so on.

It is important to note that the time series of JR_t is constructed with overlapping samples. So it is the change ΔJR_{t+1} that measures the realized jumps in month $t+1$. To test the predictability of slope on future jumps, we run the following regression:

$$\Delta JR_{t+1} = a + bs_t + \epsilon_{t+1}. \tag{13}$$

Our model implies that $b > 0$. For meaningful regression estimates, we exclude stocks with time series observations few than 24, and we end up with 806 stocks. The average estimate of b is equal to 0.037, the t-statistic of all estimates of b is equal to 2.536, and the average R^2 is 0.036. So the evidence supports our theoretical result that the slope of implied volatility smile is related to average jump size in stock returns.

[21] Jiang and Yao (2007) model jumps in stock returns using the same specification as ours. If there are jumps, then JR should be equal to $\lambda\mu_J$.

3.3 Portfolio Performance

Now, we are ready to test our main hypothesis. Panel (a) of Table 3 reports the statistics of the quintile portfolios formed on s. Most interestingly and consistent with our hypothesis, the average portfolio return in the subsequent month decreases from statistically significant 2.1% for quintile 1 to 0.2% for quintile 5, which is statistically insignificant.[22] The Sharpe ratio also decreases with slope from 0.225 to -0.008. On the other hand, the post-ranking portfolio β, which is estimated by regressing portfolio returns on returns of the S&P 500 index for the sample period, exhibits a U-shape pattern although quintile 1 has the highest β of 1.374. Returns of all quintiles except quintile 1 are negatively skewed. All quintile portfolio returns are positively autocorrelated with the highest autocorrelation coefficient of 0.133 for quintile 5.

As an unconditional test, the last row of panel (a) presents the statistics of the long-short portfolio $Q_1 - Q_5$ by long quintile 1 and short quintile 5. The average monthly return of this trading strategy is 1.8% with a t-statistic of 8.168, indicating significant statistical difference between the average returns of quintiles 1 and 5. The portfolio β is only 0.159 but its Sharpe ratio is almost three times of that for quintile 1. The time series of monthly returns is positively skewed and has high kurtosis, but is not significantly autocorrelated.

The return of the long-short portfolio $Q_1 - Q_5$ is also economically significant even in the presence of transaction costs. The annualized average return of $Q_1 - Q_5$ is 22.2%. On average, the quintile portfolios have a high turn-over rate of 73.1% per month. Assuming a 0.5% one-way transaction cost as Jegadeesh and Titman (1993), the strategy still generates an annual profit of 13.4%.[23] Given that our sample covers a more recent period, the use of 0.5% as a measure of transaction costs is very conservative.

[22] In the base case, the holding period of the portfolios starts immediately after the portfolio formation day. For robustness check, we also allow one day delay in starting the portfolio holding period. And the results with one day delay are essentially the same as those without delay.

[23] Jegadeesh and Titman (1993) cite Berkowitz, Logue, and Noser (1988) in estimating the transaction costs for the momentum trading strategies during the period of 1965 - 1989. More recent studies of the effect of transaction costs on momentum strategies include Korajczyk and Sadka (2004) and Lesmond, Schill, and Zhou (2003).

Panel (a) of Figure 1 plots monthly average slopes of the quintile portfolios. It is clear that the highest and the lowest quintiles move in opposite directions. Panel (b) plots monthly returns of the quintiles while panel (c) plots monthly returns of the long-short portfolio $Q_1 - Q_5$. The returns of the quintile portfolios move mostly together while in general quintile 1 is the highest and quintile 5 is the lowest. The return of $Q_1 - Q_5$ displays an upward trend in the first five years and a downward trend in the last four and half years. The return of $Q_1 - Q_5$ is positive in 99 of 114 months and it achieves the highest value in January, 2001. It is interesting that the lowest return of $Q_1 - Q_5$ happens in February, 2000, near the peak of the internet bubble. Around the Russian crisis and LTCM debacle in August, 1998, the return of $Q_1 - Q_5$ is close to zero but positive and remains positive for the rest of the year.

In panel (b) of Table 3, we sort stocks on s into ten deciles and report the statistics of the decile portfolios. The results are similar to those for the quintile portfolios in panel (a). In fact, as expected, the average return of the long-short portfolio $Q_1 - Q_{10}$ (2.3%) for the decile portfolios is even higher than that of $Q_1 - Q_5$ for the quintile portfolios. Because the results for decile portfolios are very similar to those for quintile portfolios, we only consider quintile portfolios for the rest of the paper.

One may wonder if the pattern of average portfolio returns can be explained by existing asset pricing models. In this paper, we consider the four-factor model of Carhart (1997) that includes the three factors ($R_M - R_f$, SMB, and HML) of Fama and French (1993) and the momentum factor (MOM) based on Jegadeesh and Titman (1993). Table 4 reports the four-factor monthly time-series regression results for the quintile portfolios formed on s. The intercept is positive and statistically significant for the lowest quintile (1.2%) , and it decreases to -0.8% and statistically significant for the highest quintile. The F-test of Gibbons, Ross, and Shanken (1989) (with p-value of 0.000%) easily rejects the null hypothesis that all the intercepts are zero. So the quintile portfolio returns cannot be explained by the four-factor model. The four-factor model does, however, capture most of time-series variation as the lowest R^2 is 0.937 for the lowest quintile. The loadings on the market and SMB factors exhibit a U-shape pattern, high for quintiles 1 and 5 but low for quintile 3. The loading on the HML factor is mostly constant across different quintiles. The loading on the MOM factor shows a hump shape as it is low for quintiles 1 and 5 but high for quintile 3.

Table 4 also reports the four-factor regression results for the long-short portfolio $Q_1 - Q_5$. The estimated coefficients are equal to the differences between those for Q_1 and Q_5. The intercept is 1.9% and highly significant. The loading on the market factor is positive but small (0.094) and only marginally significant. The loadings on the SMB and HML factors are small and statistically insignificant. The only statistically significant coefficient is that for the MOM factor, which is negative. The R^2 of the time-series regression is only 0.209, much lower than the 90%s for the quintile portfolios. In sum, the four-factor model can not explain cross-sectional average returns of the portfolios formed on s. Furthermore, the four-factor model does not capture time-series variation in the returns of the long-short portfolio Q_1-Q_5. The risk-adjusted return of Q_1-Q_5 is both statistically and economically significant. Because there is not much qualitative and quantitative difference between using raw returns and using (four-factor-model) risk-adjusted returns for all our results, we will only report raw portfolio returns in the rest of the paper for brevity.

3.4 Control for Other Explanatory Variables

Although the four-factor model cannot explain cross-sectional returns of the portfolios formed on slope of implied volatility smile, it may be the case that s is related to other stock characteristics. In this section, we examine some variables that have been found to explain cross-sectional stock returns. Particularly, we consider market β, past return, past idiosyncratic return, size, book-to-market ratio, leverage, implied volatility, idiosyncratic implied volatility, historic idiosyncratic volatility, skewness, co-skewness, systematic volatility, option volume, stock volume, and stock turnover.

Some remarks are in order for choosing these variables. The market β is estimated from a regression of stock returns on returns of the S&P 500 index using the previous two years of data.[24]. Past return r is the stock return in the month when stocks are ranked and portfolios are formed. Past idiosyncratic return defined as $r_{\text{idio}} \equiv r - \beta R_M$, where R_M is the return of the S&P 500 index during the month. Many financial variables have been shown to explain stock returns, but we only consider three of them (ME, BM, and LV) to shorten the presentation. Since slope is constructed from option implied volatilities, it is natural to examine if our results are driven by implied volatility v. Recent studies such as Goyal and Santa-Clara (2003), Ang, Hodrick, Xing, and Zhang (2006), and Spiegel and Wang (2006) have shown that idiosyncratic volatilities have explanatory power on cross-sectional stock returns. Following Dennis, Mayhew, and Stivers (2006), we define the idiosyncratic implied variance as $v_{\text{idio}}^2 \equiv v^2 - \beta^2 v_M^2$, where v_M is the implied volatility of the S&P 500 index option, also obtained from OptionMetrics.[25] We also consider the historic idiosyncratic volatility $v_{\text{idio}}^{\text{hist}}$, defined to be the standard deviation of the residuals of a market regression using previous two years of stock returns. It is well-known that positive (negative) jumps lead to positive (negative) skewness of stock returns. Harvey and Siddique (2000), while extending the unconditional skewness CAPM model of Kraus and Litzenberger (1976), find that conditional (co-)skewness helps explain the cross-sectional stock returns. We examine two measures of conditional skewness: SK, defined as the total skewness of stock returns during the last two years; and CSK, defined as the coefficient of regressing last two years stock returns on the squares of market returns, also called systematic conditional skewness. Bakshi, Kapadia, and Madan (2003) show that risk-neutral skewness is related to the option price structure. In testing the model of Bakshi, Kapadia, and Madan (2003), Duan and Wei (2007) find that it is the systematic risk proportion in the total risk that determines the risk-neutral skewness. Following Duan and Wei (2007), we define the systematic risk proportion to be $v_{\text{sys}}^2 \equiv \beta^2 v_M^2 / v^2$, and call it *systematic volatility* without ambiguity. The liquidity variables are motivated by studies like Bollen and Whaley (2004), Ofek, Richardson, and Whitelaw (2004), Pan and Poteshman (2006), and Cremers and

[24]We also used stock returns of last four years and used CRSP value-weighted index as the proxy for the market portfolio, and the results do not change quantitatively. The advantage of using the S&P 500 index is consistency because options on the index are traded and we will use the index option implied volatility to define idiosyncratic volatility and systematic volatility later.

[25]Note that this definition of the idiosyncratic variance can take negative value.

Weinbaum (2008) that document evidence of market microstructure effects on option prices and stock returns. Given the data availability, we only consider OV, SV, and TO in this paper.

We adopt two approaches to examine the explanatory power of these control variables. The first one is the double-sort methodology of Fama and French (1992) as explained in Section 3.1. We divide stocks into five quintiles by ranking on one of the control variables and then within each quintile we further divide stocks into five quintiles by ranking on s. This leads to 5×5 portfolios. If the pattern of decreasing portfolios returns in s disappears or weakens within each control variable quintile, that indicates the control variable having explanatory power for s. The second approach is the cross-sectional regression of Fama and MacBeth (1973). The advantage of this methodology is that it can incorporate multiple control variables simultaneously.

Table 5 reports the average returns of the double-sorted portfolios. The first column of each panel shows the average returns of quintile portfolios formed by ranking on each control variable alone. The overall conclusion we draw from the results of Table 5 is that none of the stock characteristics can explain the slope of implied volatility smile. In panel (a), for example, there is no clear pattern in average monthly portfolio returns when stocks are sorted on β alone. In fact, the second and third quintile portfolios have higher returns than the highest quintile portfolio. In contrast, when we sort stocks within each β quintile on s, the average portfolio returns decrease in s. The long-short portfolio $Q_1^s - Q_5^s$ formed within each β quintile earns significant positive returns, with the highest monthly return of 2.1% for the highest β quintile. So a trading strategy more profitable than our benchmark long-short strategy is long high-β/high-s stocks and short high-β/low-s stocks. From the first column of panel (b), we find that stocks with lower returns in the current month tend to earn higher returns in the next month. The difference in returns between the lowest and highest quintiles is about 0.9%. This is consistent with the short-term reversal effect documented by Jegadeesh (1990) and Lehmann (1990). Again, when we sort stocks on s within each past return quintile, we find the decreasing pattern of average portfolio returns in s. The long-short portfolio $Q_1^s - Q_5^s$ formed within the lowest r quintile generate a monthly return of 2.4%. Results in panel (c) for idiosyncratic return are similar to those in panel (b). From panels (d)-(f), the accounting variables do not capture the predictability of slope on portfolio returns while sorting on book-to-market alone leads to a slight increasing pattern in the average portfolio returns, consistent with the value effect documented in Fama and French (1992). There is a mild hump-shape pattern of returns in terms of volatility variables alone as reported in panels (g)-(i). Interestingly, the three skewness measures in panels (j)-(l) do not explain cross-sectional returns. Among the liquidity measures, only option volume (OV) seems to predict returns as the lowest OV quintile outperform the highest OV quintile by 0.6% per month. This is consist with the evidence documented in Pan and Poteshman (2006) and Cremers and Weinbaum (2008) among others that option trading is informative. But again, when we sort stocks with in each OV quintile, portfolio returns decrease in s and returns of the long-short portfolio $Q_1^s - Q_5^s$ are significantly positive.

Next, we run the Fama-MacBeth regressions and report the estimated coefficients (and t-statistics) in Table 6. Due to collinearity among the variables, we drop r_{idio}, v_{idio}^2, $v_{\text{idio}}^{\text{hist}}$, CSK, and v_{sys}^2 from this analysis. In the first regression where s is the only explanatory variable, the coefficient is negative and highly significant ($t = -9.804$) confirming our earlier findings that stock returns are decreasing in s. Then we incorporate the control variables in the regression one at a time. The coefficient for s is always significant while none of the control variables is significant. The last regression contains all the control variables. s remains significant while among the control variables only past return r is significant and ln(ME) is marginally significant. In summary, we do not find the predictability of stock returns by slope being explained by the stock characteristics considered in this paper.

4 Robustness Checks

In this section, we conduct a number of robustness checks for our findings in the previous section. Particularly, we consider seasonality, data filter rules, performance persistence, and different definitions of slope. In particular, we compare our results with those in the existing literature.

4.1 Seasonality

Table 7 presents the performance of the quintile portfolios formed on s by calender months. For some months (e.g., January), the average portfolio returns are positive while for some other months (e.g., July) they are negative. Nonetheless the monotonicity of average returns in terms of s is mostly maintained across different months. The average return of the long-short portfolio $Q_1 - Q_5$ is always positive and statistically significant except for February and September. The highest monthly return of $Q_1 - Q_5$ is achieved in January (3.7%) followed by July (3.2%). As most firms schedule quarterly earnings announcements in January, April, July, and October, there seems to be a pre-/post- earning announcement effect. That is, the negative relation between future stock return and slope of implied volatility smile is more pronounced around earnings announcements. Intuitively this makes sense as large magnitude jumps are more likely to happen in the earnings announcement season.

4.2 Different Data Filters

One concern about our findings is the choice of sample. We apply a number of different filters to repeat our exercise for subsamples and summarize the results in Table 8.

First, we exclude stocks for which s is either below -0.2 or above 0.2 to make sure our results are not driven by extreme values of s. The decreasing pattern of portfolio returns in s is the same as that for the full sample. The average monthly return of $Q_1 - Q_5$ becomes smaller (1.6%) as expected but remains highly significant. Second, we exclude financial firms and consider the subsample of 3,682 non-financial firms. The average monthly return of $Q_1 - Q_5$ for non-financial firms is actually a little higher (1.9%) than that for the full data set. Third, we only use stocks that have slope and implied volatility data for the whole sample period. For this subsample of 585 stocks, the returns of the quintile portfolios are higher than those for the full sample, possibly due to the survival bias. The middle quintile has the lowest return. But the average monthly return of $Q_1 - Q_5$ for this subsample (1.2%) is still statistically significant. Next, we check if dividends change our results because dividends affect the early exercises of American options. In the last two cases, we use stocks that either paid dividends (2,821 stocks) or that did not pay dividends (1,227 stocks) during the sample period. The results are similar to those in the base case. From the above results, it seems that our findings are robust to various data filter rules.

4.3 Performance Persistence

We next examine the persistence in returns of portfolios formed on s. Table 9 presents the portfolio performance over different holding periods up to six months. Because we use overlapping samples, the returns are serially correlated for horizons longer than one month. We calculate the t-statistics using the Newy-West procedure.

The increasing pattern of portfolio returns generally remains although the differences in average monthly returns of the quintile portfolios diminish as the holding period increases. The average monthly return of $Q_1 - Q_5$ goes down to 1% at 2-month horizon and becomes as low as 0.4% at 6-month horizon *albeit* statistically significant. In spite of some degree of persistence, most of the profit generated by the long-short trading strategy comes in the first month immediately after the portfolios being formed. This implies that jumps are short-lived. Both jump intensity and average stock jump size can all be time-varying. A stock currently with a positive average jump size may have a negative average jump size in the next month. This is exactly what we observe in the data – ranking of a stock in slope changes over time.

4.4 Different Definitions of Slope

Our definition of slope of implied volatility smile is motivated by the theoretical results of Proposition 2 and Proposition 3. In previous studies, various measures for the steepness of implied volatility smile have been proposed. In this section, we examine the portfolio performance using some of these alternative definitions.

The first variation is defined as: $\hat{s} \equiv \left(\sigma_{\text{put}}^{\text{imp}} - \sigma_{\text{call}}^{\text{imp}}\right)/v$. Because v is approximately the instantaneous stock volatility, $\hat{s}$ basically measures the percentage difference between the implied volatilities of the put and call options with $\Delta = -0.5$ and 0.5 respectively. Similar definitions have been used by, for example, Toft and Prucyk (1997) and Bollen and Whaley (2004). Different from our definition, Toft and Prucyk (1997) use the percentage difference between implied volatilities of call (put) options with strike prices 10% below and 10% above the stock price. Bollen and Whaley (2004) use the percentage difference between implied volatilities of in-the-money calls (out-of-the-money puts) and at-the-money calls (at-the-money puts) with $\Delta = 0.75$ and 0.5, respectively. The definitions of Toft and Prucyk (1997) and Bollen and Whaley (2004) measure the global steepness of implied volatility smile because the options they use have very different strike prices.

The second alternative measure we consider is also a global measure and is defined as: $sk \equiv \sigma_{\text{put}}^{\text{imp}}(-0.25) - \sigma_{\text{call}}^{\text{imp}}$, where $\sigma_{\text{put}}^{\text{imp}}(-0.25)$ is the implied volatility of the out-of-the-money put option with $\Delta = -0.25$. Zhang, Zhao, and Xing (2008) use a similar version and call it *skew*. Specifically, they use the ratio of strike price to stock price as moneyness and define skew as the difference between implied volatilities of out-of-the-money put and at-the-money call options.[26] The third alternative measure is defined as: $sl \equiv \sigma_{\text{put}}^{\text{imp}}(-0.25) - \sigma_{\text{put}}^{\text{imp}}$. This version is similar to that of Bollen and Whaley (2004) although theirs measures percentage difference. It is also similar to the *slope* variable of Zhang, Zhao, and Xing (2008) although they use put options with different strike/price ratios in stead of option deltas.

Panel (a) of Table 10 reports the average monthly returns of quintile portfolios sorted on s, $\hat{s}$, sk, and sl. The results for $\hat{s}$ are very close to those for s, showing no difference when scaling s by implied volatility. The results for sk are also similar to those for s but the average monthly return of the long-short portfolio $Q_1 - Q_5$ becomes smaller (1.5%). For sl, the decreasing

[26] Zhang, Zhao, and Xing (2008) define their *skew* by selecting options based on liquidity measures such as volume and open interest in addition to moneyness.

pattern in portfolio returns is much less significant, where the average monthly return of $Q_1 - Q_5$ is only 0.6% *albeit* statistically significant. Panel (b) presents the statistics of the four different measures of steepness. s and $\hat{s}$ are highly correlated, explaining why they lead to almost identical results. The correlation between s and sk is 0.792 while the correlation between s and sl is only 0.283.

4.5 Disentangle s, sk, and sl

By definition, s, sk, and sl satisfy the identity $s = sk - sl$. The results for sk in the previous section are similar to those documented in Zhang, Zhao, and Xing (2008). They show that high *skew* stocks underperform low *skew* by about 18% (raw return) per year, very close to our number. Zhang, Zhao, and Xing (2008) argue that their *skew* measure reflects informed investors' demand of out-of-the-money puts in anticipating bad news about future stock prices. The implication is that the option market leads the stock market and is more efficient in incorporating information. In contrast, we assume efficient stock market and option market, and our *slope* of implied volatility smile measures the jump risk of the underlying stock price. In fact, the put option used in defining s is slightly in the money. So a higher value of s cannot be interpreted as expectation of bad news. To the contrary, it should be interpreted as expectation of good news or positive jumps according to our terminology.

To further disentangle the effects of s, sk, and sl, we conduct the double-sort exercise and report the results in Table 11. In panels (a) and (b), we first sort stocks on sk and sl respectively. Then we sort on s within each sk and sl quintile. sk seems to have some limited explanatory power on s as the decreasing pattern in average portfolio returns within each sk quintile becomes less significant than that without first sorting on sk. But the average return of $Q_1^s - Q_5^s$ is still large (as high as 1.6% per month for the lowest and highest skew quintiles) and significant across sk quintiles. sl is not capable of explaining s at all as the return of $Q_1^s - Q_5^s$ is always large and significant across sl quintiles. In panels (c) and (d), we first sort stocks on s and then sort stocks on sk and sl respectively within each s quintile. Panel (c) shows that the decreasing pattern of portfolio returns in sk becomes very flat or disappears within the s quintiles. The return $Q_1^{sk}-Q_5^{sk}$ becomes negative for the second highest s quintile. However, some predictability of sk remains as the return of $Q_1^{sk}-Q_5^{sk}$ is significant for the lowest and highest s quintiles. As shown in panel (d), sl's predictability disappears once we control for s. The return of $Q_1^{sl}-Q_5^{sl}$ is always small and insignificant across s quintiles. In summary, our findings indicate that the predictability of s is not driven by sk and sl while the predictability of sk and sl can be explained to a large extent by s.

It is important to point out that our slope measure is different from the deviation measures of put-call parity examined by Ofek, Richardson, and Whitelaw (2004), Cremers and Weinbaum (2008), and Zhang, Zhao, and Xing (2008). These papers either use option price or implied volatility to measure deviations from put-call parity for options with the same strike price. The put and call options in our slope variable s do not have the same strike price for two reasons. First, these options are hypothetical - they are interpolated from observed option contracts. Second, even for non-dividend-paying stocks, the strike prices of put option with $\Delta=-0.5$ and call option with $\Delta=0.5$ are not the same due to early exercise opportunities. Despite these differences, there are similarity in the results of these papers and ours. For example, Cremers and Weinbaum (2008) find that stocks with relatively expensive calls outperform stocks with relatively expensive puts by 50 basis points per week. This is consistent with our findings if we interprete lower value of s as relative expensive call option and higher value of s as relatively expensive put option. Again, these papers attribute the predictability of stock returns by deviations from put-call parity to limits of arbitrage or mispricing in stock/option markets. In contrast, we assume that markets are efficient and the slope of implied volatility smile just reflects the jump risk.

5 Conclusion

Strong empirical evidence suggests that stock prices contain jumps. We present a very general jump-diffusion model for the SDF and stock price processes and show that expected stock return is related to the jump risk. We argue that the average SDF jump size is positive and hence the stock return is decreasing in the average stock jump size. To overcome the difficulty of estimating jump distribution, we show that the average stock jump size can be inferred from the observed slope of option implied volatility smile. We obtain the testable hypothesis that stock portfolios with low (high) slopes have high (low).

We empirically test the hypothesis using individual equity option data and find strong supporting evidence. The future returns of stock portfolios formed on slope show an decreasing pattern in slope. The trading strategy that long the lowest slope quintile portfolio and short the highest slope quintile portfolio generates annual profit of 22.2%. Our findings cannot be explained by the four-factor model that includes the three Fama-French factors and momentum factor. Our results also hold after controlling for a number of stock characteristics.

Appendix

In this appendix, we provide technical details for the theoretical results.

Proof of Proposition 1: It is more convenient to work with independent Brownian motions and Poisson processes. The Brownian motions are easy to decompose as we can find $\widetilde{W_i}$ independent of W_M such that $W_i = \rho_i W_M + \sqrt{1-\rho_i^2}\widetilde{W_i}$. Intuitively, $\widetilde{W_i}$ is the idiosyncratic diffusive component in the stock price. For the Poisson processes, we use the following decompositions $N_M = N_C + \widetilde{N}_M$ and $N_i = N_C + \widetilde{N}_i$ where N_C, $\widetilde{N}_M$, and $\widetilde{N}_i$ are independent Poisson processes with intensities λ_C, $\widetilde{\lambda}_M$, and $\widetilde{\lambda}_i$ respectively. Here, N_C is the common factor that drives both N_M and N_i while $\widetilde{N}_i$ is the idiosyncratic jump component in the stock price. Direct calculation shows that $\mathrm{Corr}(N_M, N_i) = \frac{\lambda_C}{\sqrt{\lambda_M \lambda_i}}$. So we have $\lambda_C = \eta_i \sqrt{\lambda_M \lambda_i}$, $\widetilde{\lambda}_M = \lambda_M - \lambda_C$, and $\widetilde{\lambda}_i = \lambda_i - \lambda_C$. We assume that λ_M, λ_i, and η_i satisfy the constraints $\widetilde{\lambda}_M \geq 0$ and $\widetilde{\lambda}_i \geq 0$.

Substituting the above decompositions into (2) and (4), we obtain alternative representation of the SDF and stock price processes in terms of independent Brownian motions and Poisson processes. Using the new representation and applying the Itô's formula for jump-diffusions (see, for example, Protter (2004)) to the product MS_i, we get:

$$\begin{aligned} \frac{d(MS_i)}{MS_i} &= (\mu_i - r_f + \rho_i \sigma_M \sigma_i - \lambda_M \mu_{J_M} - \lambda_i \mu_{J_i})\, dt + \sigma_M dW_M + \sigma_i dW_i \\ &\quad + J_M dN_M + J_i dN_i + J_M J_i dN_C. \end{aligned} \tag{A.1}$$

MS_i being a martingale (as in equation (1)) implies:

$$\mu_i - r_f + \rho_i \sigma_M \sigma_i + \eta_i \sqrt{\lambda_M \lambda_i} \mathrm{E}[J_M J_i] = 0. \tag{A.2}$$

We can rewrite $J_M J_i = (1 + J_M)(1 + J_i) - J_M - J_i - 1$. We know from equations (3) and (5) that $1 + J_M$ and $1 + J_i$ are log-normally distributed with correlation coefficient ψ_i. Then direct computation of the above expectation leads to (6). The monotonicity of excess stock return in (i) - (iii) can be easily derived by differentiating the right hand side of (6) with respect to the corresponding parameters.

Proof of Proposition 2: We assume that under the risk-neutral probability measure, the stock price process (4) can be rewritten as:

$$dS_i/S_i = (r_f - q_i - \lambda_i \mu_{J_i})\, dt + \sigma_i dW_i + J_i dN_i. \tag{A.3}$$

To be rigorous, the jump intensity and jump size distribution have to be modified when we switch from the objective probability measure to the risk-neutral probability measure. Technically, we should use λ_i^*, $\mu_{J_i}^*$, and $\sigma_{J_i}^*$ to denote the jump intensity, average jump size, and jump volatility under the risk-neutral probability measure. Because the market is incomplete in the presence of jumps, the transformation between the two probability measures is not unique. Santa-Clara and Yan (2008), for example, find a transformation for their equilibrium model, which depends on the risk aversion of the representative investor. We abuse the notation here by using the same parameters for two different probability measures. However, ignoring the change of probability measure may not be a serious problem because the same transformation applies to all stocks. As we consider cross-sectional stock returns, the probability transformation won't change our inference qualitatively.

Under the risk-neutral probability measure, the price (C) of a call option on the i-th stock is equal to the discounted expected payoff:

$$C = e^{-r_f T} \mathrm{E}_0 \left[(S_i(T) - K)^+ \right], \tag{A.4}$$

where $\mathrm{E}_0(.)$ denotes the expectation. We consider very short time-to-maturity, i.e., T is small. For the Poisson process, the probability that one jump occurs before T is λT while the probability of multiple jumps is of order $O(T^2)$. So up to order of T^2, the log terminal stock price can be approximated by the mixture of normal distributions:

$$\ln S_i(T) = \begin{cases} \ln S_i(0) + \left(r_f - q_i - \frac{1}{2}\sigma_i^2 - \lambda_i \mu_{J_i}\right) T + \sigma_i \sqrt{T} \epsilon & \text{w/ Prob. } 1 - \lambda_i T \\ \ln S_i(0) + \left(r_f - q_i - \frac{1}{2}\sigma_i^2 - \lambda_i \mu_{J_i}\right) T + \sigma_i \sqrt{T} \epsilon + (\mu_{J_i} + \sigma_{J_i} \zeta) & \text{w/ Prob. } \lambda_i T \end{cases},$$

where ϵ and ζ are standard normally distributed variables independent of each other. We can then rewrite the option price (A.4) as:

$$C = I_1 + I_2, \tag{A.5}$$

where I_1 and I_2 correspond to the components without and with jump respectively. Letting $\Phi(.)$ represent the cdf of the standard normal distribution, direct computation shows:

$$\begin{aligned}
I_1 &= (1-\lambda_i T)\left[S_i(0)e^{-\left(q_i+\lambda_i\mu_{J_i}\right)T}\Phi\left(\frac{\ln(S_i(0)/K)+\left(r_f-q_i-\lambda_i\mu_{J_i}+\frac{1}{2}\sigma_i^2\right)T}{\sigma_i\sqrt{T}}\right)\right.\\
&\quad \left. -Ke^{-r_fT}\Phi\left(\frac{\ln(S_i(0)/K)+\left(r_f-q_i-\lambda_i\mu_{J_i}-\frac{1}{2}\sigma_i^2\right)T}{\sigma_i\sqrt{T}}\right)\right],\\
I_2 &= \lambda_i T\left[S_i(0)e^{-\left(q_i+\lambda_i\mu_{J_i}\right)T}\Phi\left(\frac{\ln(S_i(0)/K)+\left(r_f-q_i-\lambda_i\mu_{J_i}+\frac{1}{2}\sigma_i^2\right)T+\mu_{J_i}+\sigma_{J_i}^2}{\sqrt{\sigma_i^2T+\sigma_{J_i}^2}}\right)\right.\\
&\quad \left. -Ke^{-r_fT-\mu_{J_i}-\frac{1}{2}\sigma_{J_i}^2}\Phi\left(\frac{\ln(S_i(0)/K)+\left(r_f-q_i-\lambda_i\mu_{J_i}-\frac{1}{2}\sigma_i^2\right)T+\mu_{J_i}}{\sqrt{\sigma_i^2T+\sigma_{J_i}^2}}\right)\right].
\end{aligned}$$

We rewrite these equations in terms of log moneyness X:

$$\begin{aligned}
I_1 &= (1-\lambda_i T)S_i(0)e^{-q_iT}\left[e^{-\lambda_i\mu_{J_i}T}\Phi\left(\frac{-X+\left(-\lambda_i\mu_{J_i}+\frac{1}{2}\sigma_i^2\right)T}{\sigma_i\sqrt{T}}\right)\right.\\
&\quad \left. -e^X\Phi\left(\frac{-X-\left(\lambda_i\mu_{J_i}+\frac{1}{2}\sigma_i^2\right)T}{\sigma_i\sqrt{T}}\right)\right],\\
I_2 &= \lambda_i T S_i(0)e^{-q_iT}\left[e^{-\lambda_i\mu_{J_i}T}\Phi\left(\frac{-X+\left(-\lambda_i\mu_{J_i}+\frac{1}{2}\sigma_i^2\right)T+\mu_{J_i}+\sigma_{J_i}^2}{\sqrt{\sigma_i^2T+\sigma_{J_i}^2}}\right)\right.\\
&\quad \left. -e^Xe^{-\mu_{J_i}-\frac{1}{2}\sigma_{J_i}^2}\Phi\left(\frac{-X-\left(\lambda_i\mu_{J_i}+\frac{1}{2}\sigma_i^2\right)T+\mu_{J_i}}{\sqrt{\sigma_i^2T+\sigma_{J_i}^2}}\right)\right].
\end{aligned}$$

Substituting the above identities into (A.5) and ignoring higher order terms (with Taylor approximation $e^z = 1 + z + O(z^2)$), we get:

$$C = S_i(0)\left[\Phi\left(\frac{-X+\left(-\lambda_i\mu_{J_i}+\frac{1}{2}\sigma_i^2\right)T}{\sigma_i\sqrt{T}}\right)-e^X\Phi\left(\frac{-X+\left(-\lambda_i\mu_{J_i}-\frac{1}{2}\sigma_i^2\right)T}{\sigma_i\sqrt{T}}\right)\right]+O(T). \tag{A.6}$$

Letting $X = 0$ and applying the Taylor expansion of Φ around zero ($\Phi(z) = \frac{1}{2} + \frac{z}{\sqrt{2\pi}} + O(z^2)$), equation (A.6) becomes:

$$\begin{aligned} C|_{X=0} &= S_i(0)\left[\frac{1}{2} + \frac{1}{\sqrt{2\pi}}\left(-\lambda_i\mu_{J_i} + \frac{1}{2}\sigma_i^2\right)\sqrt{T} - \frac{1}{2} + \frac{1}{\sqrt{2\pi}}\left(\lambda_i\mu_{J_i} + \frac{1}{2}\sigma_i^2\right)\sqrt{T}\right] + O(T) \\ &= \frac{1}{\sqrt{2\pi}}S_i(0)\sigma_i\sqrt{T} + O(T). \end{aligned} \tag{A.7}$$

We are also interested in the derivative of C with respect to X. Differentiating $C = I_1 + I_2$ with respect to X, we get:[27]

$$\begin{aligned} \frac{\partial C}{\partial X} &= (1-\lambda_i T)S_i(0)e^{-q_iT}\left\{-\frac{e^{-\lambda_i\mu_{J_i}T}}{\sigma_i\sqrt{T}}\phi\left(\frac{-X + \left(-\lambda_i\mu_{J_i} + \frac{1}{2}\sigma_i^2\right)T}{\sigma_i\sqrt{T}}\right)\right. \\ &\left. -e^X\left[\Phi\left(\frac{-X - \left(\lambda_i\mu_{J_i} + \frac{1}{2}\sigma_i^2\right)T}{\sigma_i\sqrt{T}}\right) - \frac{1}{\sigma_i\sqrt{T}}\phi\left(\frac{-X - \left(\lambda_i\mu_{J_i} + \frac{1}{2}\sigma_i^2\right)T}{\sigma_i\sqrt{T}}\right)\right]\right\} + \frac{\partial I_2}{\partial X}, \end{aligned}$$

where $\phi(z) \equiv \frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$ is the density function of standard normal distribution. It turns out that the derivative of I_2 is of order $O(T)$ for $X = 0$. So we do not show it explicitly for brevity. Evaluating the above equation at $X = 0$, we find:

$$\begin{aligned} \left.\frac{\partial C}{\partial X}\right|_{X=0} = \; & S_i(0)\left\{-\frac{e^{-\lambda_i\mu_{J_i}T}}{\sigma_i\sqrt{T}}\phi\left(\frac{\left(-\lambda_i\mu_{J_i} + \frac{1}{2}\sigma_i^2\right)\sqrt{T}}{\sigma_i}\right) - \left[\Phi\left(\frac{-\left(\lambda_i\mu_{J_i} + \frac{1}{2}\sigma_i^2\right)\sqrt{T}}{\sigma_i}\right)\right.\right. \\ & \left.\left. -\frac{1}{\sigma_i\sqrt{T}}\phi\left(\frac{-\left(\lambda_i\mu_{J_i} + \frac{1}{2}\sigma_i^2\right)\sqrt{T}}{\sigma_i}\right)\right]\right\} + O(T), \end{aligned} \tag{A.8}$$

Applying Taylor approximations for e^z, ϕ, and Φ around zero ($e^z = 1 + z + O(z^2)$, $\phi(z) = \frac{1}{\sqrt{2\pi}}\left(1 - \frac{z^2}{2}\right) + O(z^4)$) leads to:

$$\left.\frac{\partial C}{\partial X}\right|_{X=0} = S_i(0)\left[-\frac{1}{2} + \frac{1}{\sqrt{8\pi}}\left(\sigma_i + \frac{2\lambda_i\mu_{J_i}}{\sigma_i}\right)\sqrt{T}\right] + O(T). \tag{A.9}$$

Next, we compute the option price and the derivative of the option price in terms of moneyness using an alternative method. Let C^{BS} denote the option value derived from the Black-Scholes formula using some implied volatility function. We define $\sigma_i^{\text{imp}}(X, T)$ so that $C = C^{\text{BS}}$, that is,

[27]It is incorrect to differentiate (A.6).

the Black-Scholes formula correctly prices the option using the appropriate implied volatility function. We assume $\sigma_i^{\text{imp}}(X,T)$ to be differentiable at $X=0$. Note that C^{BS} is a function of the implied volatility, which is also a function of moneyness:

$$C = C^{\text{BS}} = S_i(0)e^{-q_iT}\Phi(d_1) - Ke^{-r_fT}\Phi(d_2) = S_i(0)e^{-q_iT}\left[\Phi(d_1) - e^X\Phi(d_2)\right],$$

where $d_1 = \frac{-X+\frac{1}{2}\left(\sigma_i^{\text{imp}}\right)^2 T}{\sigma_i^{\text{imp}}\sqrt{T}}$ and $d_2 = \frac{-X-\frac{1}{2}\left(\sigma_i^{\text{imp}}\right)^2 T}{\sigma_i^{\text{imp}}\sqrt{T}}$. Letting $X=0$ (so that $d_1 = \frac{1}{2}\sigma_i^{\text{imp}}\sqrt{T}$ and $d_2 = -\frac{1}{2}\sigma_i^{\text{imp}}\sqrt{T}$) and using the Taylor expansion of Φ, we have:

$$C|_{X=0} = \frac{1}{\sqrt{2\pi}}S_i(0)\sigma_i^{\text{imp}}\sqrt{T} + O(T). \tag{A.10}$$

We now consider derivative of C with respect to X. By the chain rule,

$$\frac{\partial C}{\partial X} = \frac{\partial C^{\text{BS}}}{\partial X} + \frac{\partial C^{\text{BS}}}{\partial \sigma_i^{\text{imp}}}\frac{\partial \sigma_i^{\text{imp}}}{\partial X}. \tag{A.11}$$

We use the Black-Scholes formula to evaluate the above expression to get:

$$\frac{\partial C}{\partial X} = S_i(0)e^{-q_iT}\left\{-e^X\Phi(d_2) + \left[\phi(d_1)\frac{X+\frac{1}{2}\left(\sigma_i^{\text{imp}}\right)^2 T}{\left(\sigma_i^{\text{imp}}\right)^2\sqrt{T}} - e^X\phi(d_2)\frac{X-\frac{1}{2}\left(\sigma_i^{\text{imp}}\right)^2 T}{\left(\sigma_i^{\text{imp}}\right)^2\sqrt{T}}\right]\frac{\partial \sigma_i^{\text{imp}}}{\partial X}\right\}.$$

By setting $X=0$ and applying Taylor approximations for Φ and ϕ, we have:

$$\left.\frac{\partial C}{\partial X}\right|_{X=0} = S_i(0)\left[-\frac{1}{2} + \frac{1}{\sqrt{8\pi}}\left(\sigma_i^{\text{imp}} + \frac{2\partial\sigma_i^{\text{imp}}}{\partial X}\right)\sqrt{T}\right] + O(T). \tag{A.12}$$

Respectively comparing (A.7) with (A.10) and (A.9) with (A.12), we derive equations (13) and (14), and thus prove Proposition 1.

Proof of Proposition 3: Again we assume T small. Consider the implied volatilities $\sigma_{i,\text{put}}^{\text{imp}}$ and $\sigma_{i,\text{call}}^{\text{imp}}$ for the put and call options with $\Delta_{\text{put}} = -0.5$ and $\Delta_{\text{call}} = 0.5$ respectively. Let X_{put} and

X_{call} be their corresponding log moneyness. From the Black-Scholes formula, we have:

$$\Delta_{\text{put}} = e^{-q_i T}\left[\Phi(d_{1,\text{put}}) - 1\right] \quad \text{and} \quad \Delta_{\text{call}} = e^{-q_i T}\Phi(d_{1,\text{call}}), \quad \text{where}$$

$$d_{1,\text{put}} = \frac{-X_{\text{put}} + \frac{1}{2}\left(\sigma_{i,\text{put}}^{\text{imp}}\right)^2 T}{\sigma_{i,\text{put}}^{\text{imp}}\sqrt{T}} \quad \text{and} \quad d_{1,\text{call}} = \frac{-X_{\text{call}} + \frac{1}{2}\left(\sigma_{i,\text{call}}^{\text{imp}}\right)^2 T}{\sigma_{i,\text{call}}^{\text{imp}}\sqrt{T}}.$$

By the fact that $\Delta_{\text{put}} = -0.5$ and $\Delta_{\text{call}} = 0.5$, we get:

$$\Phi(d_{1,\text{put}}) = 1 - 0.5e^{q_i T} \quad \text{and} \quad \Phi(d_{1,\text{call}}) = 0.5e^{q_i T}.$$

Using the Taylor approximation of Φ ($\Phi(d_{1,\text{put}}) \approx \frac{1}{2} + \frac{d_{1,\text{put}}}{\sqrt{2\pi}}$ and $\Phi(d_{1,\text{call}}) \approx \frac{1}{2} + \frac{d_{1,\text{call}}}{\sqrt{2\pi}}$),[28] we have:

$$d_{1,\text{put}} \approx \sqrt{\frac{\pi}{2}}\left(1 - e^{q_i T}\right), \tag{A.13}$$

$$d_{1,\text{call}} \approx \sqrt{\frac{\pi}{2}}\left(e^{q_i T} - 1\right). \tag{A.14}$$

Now applying the Taylor approximation of the exponential function ($e^{q_i T} \approx 1 + q_i T$), we get:

$$d_{1,\text{put}} = O(T) \quad \text{and} \quad d_{1,\text{call}} = O(T).$$

Then from the definitions of $d_{1,\text{put}}$ and $d_{1,\text{call}}$, we get:

$$-X_{\text{put}} + \frac{1}{2}\left(\sigma_{i,\text{put}}^{\text{imp}}\right)^2 T = O(T^{3/2}) \quad \text{and} \quad -X_{\text{call}} + \frac{1}{2}\left(\sigma_{i,\text{call}}^{\text{imp}}\right)^2 T = O(T^{3/2}).$$

Therefore,

$$X_{\text{put}} = \frac{1}{2}\left(\sigma_{i,\text{put}}^{\text{imp}}\right)^2 T + O(T^{3/2}) = O(T), \tag{A.15}$$

$$X_{\text{call}} = \frac{1}{2}\left(\sigma_{i,\text{call}}^{\text{imp}}\right)^2 T + O(T^{3/2}) = O(T). \tag{A.16}$$

[28] The approximation errors are up to order of $O(d_1^3)$. As we see next, these approximations are of order $O(T^3)$ because d_1 is of order $O(T)$.

The above two equations imply that both put and call options are very close to (in the order of T) being at-the-money. Assuming bounded derivative for the implied volatility function, then the implied volatilities of the put and call options are all close to the instantaneous stock volatility (using equation (13) and Taylor approximation $\sigma_i^{\text{imp}}(z) = \sigma_i^{\text{imp}}(0) + \frac{\partial \sigma_i^{\text{imp}}}{\partial X}(0) z + O(z^2)$). In fact, we have:

$$\begin{aligned} \sigma_{i,\text{put}}^{\text{imp}} &= \sigma_i + O(T), && \text{(A.17)} \\ \sigma_{i,\text{call}}^{\text{imp}} &= \sigma_i + O(T). && \text{(A.18)} \end{aligned}$$

Combining these two equations proves equation (11).

To prove equation (12), we first plug equations (A.17) and (A.18) into equations (A.15) and (A.16) to find:

$$\begin{aligned} X_{\text{put}} &= \frac{1}{2}\sigma_i^2 T + O(T^{3/2}), && \text{(A.19)} \\ X_{\text{call}} &= \frac{1}{2}\sigma_i^2 T + O(T^{3/2}). && \text{(A.20)} \end{aligned}$$

Use approximations (A.13) and (A.14) and take the difference between $d_{1,\text{put}}$ and $d_{1,\text{call}}$ to get:

$$\frac{-X_{\text{put}} + \frac{1}{2}\left(\sigma_{i,\text{put}}^{\text{imp}}\right)^2 T}{\sigma_{i,\text{put}}^{\text{imp}}\sqrt{T}} - \frac{-X_{\text{call}} + \frac{1}{2}\left(\sigma_{i,\text{call}}^{\text{imp}}\right)^2 T}{\sigma_{i,\text{call}}^{\text{imp}}\sqrt{T}} \approx \sqrt{2\pi}\left(1 - e^{q_i T}\right).$$

The above equation can be rewritten as:

$$X_{\text{put}} - X_{\text{call}} \approx \sqrt{2\pi}\left(e^{q_i T} - 1\right)\sigma_{i,\text{put}}^{\text{imp}}\sqrt{T} + \frac{1}{2}(\sigma_{i,\text{put}}^{\text{imp}} - \sigma_{i,\text{call}}^{\text{imp}})\sigma_{i,\text{put}}^{\text{imp}} T + \frac{(\sigma_{i,\text{put}}^{\text{imp}} - \sigma_{i,\text{call}}^{\text{imp}})X_{\text{call}}}{\sigma_{i,\text{call}}^{\text{imp}}}.$$

Note that from (A.19) and (A.20), $\sigma_{i,\text{put}}^{\text{imp}} - \sigma_{i,\text{call}}^{\text{imp}}$ is of order $O(T^{3/2})$ and X_{call} is of order $O(T)$. $\sigma_{i,\text{put}}^{\text{imp}}$ can be approximated by $v_i = 0.5(\sigma_{i,\text{call}}^{\text{imp}} + \sigma_{i,\text{put}}^{\text{imp}})$ up to order $O(T)$. So we can drop the last two terms in the above equation which are of order $O(T^2)$, and have the following approximation:

$$X_{\text{put}} - X_{\text{call}} \approx \sqrt{2\pi}\left(e^{q_i T} - 1\right) v_i \sqrt{T}. \qquad \text{(A.21)}$$

Note that the value of (A.21) is non zero only when the dividend yield q_i is non-zero. If that is the case, we can approximate the slope of the implied volatility smile by:[29]

$$\left.\frac{\partial \sigma_i^{\text{imp}}(X,T)}{\partial X}\right|_{X=0} \approx \frac{\sigma_{i,\text{put}}^{\text{imp}} - \sigma_{i,\text{call}}^{\text{imp}}}{X_{\text{put}} - X_{\text{call}}} = \frac{\sigma_{i,\text{put}}^{\text{imp}} - \sigma_{i,\text{call}}^{\text{imp}}}{\sqrt{2\pi}\,(e^{q_i T} - 1)\, v_i \sqrt{T}}. \tag{A.22}$$

Using the approximation $v_i \approx \sigma_i$ and comparing (A.22) and (8), we see that s_i is proportional to $\lambda_i \mu_{J_i}$ up to the constant $L_i = 2\sqrt{2\pi T}\left(e^{q_i T} - 1\right)$. And this proves (12).

It should be pointed out that our results depend on the assumption of non-zero dividend yield. For a non-dividend-paying stock, the European put and call with $\Delta = -0.5$ and 0.5 have the same strike price and thus the same implied volatility to exclude arbitrage opportunity. Hence the slope defined in this paper is zero in this case. However, the traded options are American style. Even for non-dividend-paying stocks, the put and call options with $\Delta = -0.5$ and 0.5 can have different strikes because of early exercise opportunities. Our empirical analysis shows that our results are not affected by stock dividends. We leave generalizing the above results to American options for future research.

Monte-Carlo Simulations

In this section, we conduct Monte-Carlo simulations to examine the approximation errors in Proposition 2. We extend the model to incorporate stochastic volatility because of overwhelming empirical evidence of time-varying volatility. In particular, we let stock return volatility follow the square-root model of Heston (1993):

$$d\sigma_i^2 = \kappa_i(\theta_i - \sigma_i^2)dt + \phi_i \sqrt{\sigma_i^2} dZ_i, \tag{A.23}$$

where Z_i is a standard Brownian motion correlated with W_i and the correlation coefficient is $\text{Corr}(dW_i, dZ_i) = \zeta_i$. Although semi-analytical option pricing formula is available for jump-diffusion model of (4), (5), and (A.23) (see, for example, Pan (2002)), we adopt the simulation

[29] To see the first approximation, note that for a twice differentiable function, $\frac{f(x_1)-f(x_2)}{x_1-x2} = f'(x_1) + O(x_1 - x_2) = f'(x_0) + O(x_0 - x_1) + O(x_1 - x_2)$.

approach here to compute option prices because of its simplicity. To simulate paths of stock prices, we use the Euler scheme to discretize the continuous-time model and choose the time interval Δt to be 1/5 of a day. We approximate the Poisson process by a Bernoulli process, that is, there is at most one jump during an interval.

In the benchmark case, we use the following parameter values: $r_f = 0.06$, $q_i = 0.02$, $\lambda_i = 0.5$, $\mu_{J_i} = 0.1$, $\sigma_{J_i} = 0.1$, $\kappa_i = 0.02$, $\theta_i = 0.025$, $\phi_i = 0.025$, $\zeta_i = -0.25$. We set the initial stock price $S(0) = \$40$ and the initial volatility $\sigma_i(0) = 0.5$. We generate 1,000,000 paths of stock prices and compute the price of an option by taking the average pay-off as in (A.4). We then invert the Black-Scholes formula to get the option implied volatility. For a particular maturity T, we compute the at-the-money implied volatility for which the moneyness X is zero. We also compute the implied volatility for the option with the same maturity but with strike price \$0.001 higher than the strike price of the at-the-money option. We approximate the slope by the ratio between the difference of the two implied volatilities and the difference between the two moneyness values. We consider four different maturities: one day, one week, one month, and two months. We also consider the effects of changing certain parameter values, and report the at-the-money implied volatility and slope in Table A.1.

In panel (a), we use different values of average jump size, μ_{J_i}. The left half of the panel reports the implied volatility. For a fixed value of T, we see a U-shaped pattern of implied volatility as a function of μ_{J_i}. The implied volatility is biased upward, that is, higher than the instantaneous diffusive volatility, which is equal to 0.5. The bias is very small for maturity of one day but becomes larger for long maturities. For example, at two months horizon and when $\mu_{J_i} = 0.2$, the implied volatility error is 0.04. As expected, the estimated slope of implied volatility smile shows an increasing pattern in terms of μ_{J_i} when T is fixed. The rate of increase is highest when T is one day and it gets smaller as T becomes larger. To get a sense of the accuracy of the approximation, we compare slope with μ_{J_i} as equation (8) suggests that these two quantities should be close because of our choice of $\lambda_i = \sigma_i(0) = 0.5$. When $\mu_{J_i} = 0.2$ and T is 1 day, slope is 0.143, so the error is -0.057. For $\mu_{J_i} = 0.1$, the error is -0.045. The magnitude of error is smaller for negative jump sizes. For example, for

$\mu_{J_i} = -0.1$, the bias is only 0.002. Fixing a value of μ_{J_i}, the error is increasing with T and becomes quite significant, particularly for positive values of μ_{J_i}. Overall, the approximation error is significant when average jump size is positive and/or when maturity is long. However, it is important to notice that the slope of implied volatility maintains an increasing pattern in terms of μ_{J_i}. The implication is that high slope stocks have more positive jumps than low slope stocks. This is exactly what we need to formulate our main hypothesis of the paper.

In panel (b), we examine the effect of jump intensity, λ_i. As λ_i increases, the error in implied volatility becomes larger but still relatively small in magnitude. For values of λ_i equal to 1 and 2, we should compare μ_{J_i} with half of and quarter of slope. As λ_i increases, the approximation error decreases.

In panel (c), we examine the effect of correlation between the stock and volatility processes, ζ_i. The error in implied volatility is not affected by ζ_i, while the error in slope becomes smaller for higher values of ζ_i although the improvements are small.

Our most general model includes Poisson jump and stochastic volatility, and we call it the SV-J model. When there is no jump and volatility is constant ($\lambda_i = 0, \phi_i = 0$), it becomes Black-Scholes' Geometric Brownian motion (GB) model. When volatility is stochastic but there is no jumps ($\lambda_i = 0$), the model becomes Heston's (SV) model. In the case of constant volatility ($\phi_i = 0$), it becomes Merton's jump-diffusion (GB-J) model. Panel (d) of Table A.1 reports the implied volatility and slope for these different models. For implied volatility, the approximation error is larger for the models with jumps. But the magnitude of errors is small. When jumps are absent, slope is negative and small. In contrast, for the SV-J and GB-J models, slope is positive at least for short maturities. To summarize, slope is related to jumps and not affected much by stochastic volatility.

Table A.1 **Implied Volatility and Slope from Monte-Carlo Simulations**

	$\sigma_i^{\text{imp}}(X,T)\|_{X=0}$				$\frac{\partial \sigma_i^{\text{imp}}(X,T)}{\partial X}\|_{X=0}$			
T	1 day	1 week	1 month	2 months	1 day	1 week	1 month	2 months
μ_{J_i}				(a)				
0.2	0.508	0.519	0.532	0.540	0.143	0.094	0.025	−0.014
0.1	0.504	0.509	0.515	0.519	0.055	0.027	−0.011	−0.029
0.05	0.503	0.506	0.510	0.513	0.014	0.002	−0.021	−0.032
−0.05	0.502	0.505	0.508	0.511	−0.059	−0.042	−0.042	−0.042
−0.1	0.503	0.507	0.511	0.515	−0.098	−0.068	−0.054	−0.052
−0.2	0.506	0.514	0.523	0.529	−0.188	−0.146	−0.100	−0.088
λ_i				(b)				
0.5	0.504	0.509	0.515	0.519	0.055	0.027	−0.011	−0.029
1	0.508	0.517	0.528	0.533	0.129	0.073	0.007	−0.024
2	0.516	0.534	0.552	0.562	0.280	0.160	0.035	−0.022
ζ_i				(c)				
−0.5	0.504	0.509	0.515	0.519	0.053	0.024	−0.014	−0.032
−0.25	0.504	0.509	0.515	0.519	0.055	0.027	−0.011	−0.029
0	0.504	0.509	0.515	0.519	0.058	0.030	−0.008	−0.026
0.25	0.504	0.509	0.515	0.519	0.060	0.033	−0.004	−0.023
0.5	0.504	0.509	0.515	0.519	0.063	0.036	−0.001	−0.020
Model				(d)				
GB	0.500	0.501	0.502	0.505	−0.018	−0.017	−0.024	−0.029
SV	0.500	0.501	0.502	0.505	−0.020	−0.020	−0.027	−0.032
GB-J	0.504	0.509	0.515	0.520	0.058	0.029	−0.007	−0.026
SV-J	0.504	0.509	0.515	0.519	0.055	0.027	−0.011	−0.029

References

Ait-Sahalia, Yacine, 2004, Disentangling diffusion from jumps, *Journal of Financial Economics* 74, 487–528.

Anderson, Torben, Luca Benzoni, and Jesper Lund, 2002, An empirical investigation of continuous-time equity return models, *Journal of Finance* 57, 1239–1284.

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang, 2006, The cross-section of volatility and expected returns, *Journal of Finance* 61, 259–299.

Bakshi, Gurdip, Charles Cao, and Zhiwu Chen, 1997, Empirical performance of alternative option pricing models, *Journal of Finance* 52, 589–667.

Bakshi, Gurdip, Nikunj Kapadia, and Dilip Madan, 2003, Stock returns characteristics, skew laws, and the differential pricing of individual equity options, *Review of Financial Studies* 16, 101–143.

Ball, Clifford A., and Walter N. Torous, 1983, A simplified jump process for common stock returns, *Journal of Financial and Quantitative Analysis* 18, 53–65.

——— , 1985, On jumps in common stock prices and their impact on call option pricing, *Journal of Finance* 40, 155–173.

Banz, Rolf W., 1981, The relationship between return and market value of common stocks, *Journal of Financial Economics* 9, 3–18.

Barro, Robert J., 2006, Rare disasters and asset markets in the twentieth century, *Quarterly Journal of Economics* 121, 823–866.

Basu, Sanjoy, 1983, The relationship between earning yield, market value, and return for NYSE common stocks: Further evidence, *Journal of Financial Economics* 12, 129–156.

Bates, David, 1996, Jumps and stochastic volatility: Exchange rate processes implicit in Deutsche Mark options, *Review of Financial Studies* 9, 69–107.

———, 2000, Post-'87 crash fears in the S&P 500 futures option market, *Journal of Econometrics* 94, 181–238.

Berkowitz, Stephen A., Dennis E. Logue, and Eugene A. Noser, 1988, The total costs of transactions on the NYSE, *Journal of Finance* 43, 97–112.

Black, Fischer, 1976, Studies in stock price volatility changes, in *Proceedings of American Statistical Association, Business and Economic Statistics Section* pp. 177–181.

———, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637–654.

Bollen, Nicolas P. B., and Robert E. Whaley, 2004, Does net buying pressure affect the shape of implied volatility functions?, *Journal of Finance* 59, 711–753.

Breeden, Douglas T., 1979, An intertemporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics* 7, 265–296.

Carhart, Mark M., 1997, On persistence in mutual fund performance, *Journal of Finance* 52, 57–82.

Carr, Peter, and Liuren Wu, 2003, What type of process underlies options? A simple robust test, *Journal of Finance* 58, 2581–2610.

Chernov, Mikhael, A. Ronald Gallant, Eric Ghysels, and George Tauchen, 2003, Alternative models for stock price dynamics, *Journal of Econometrics* 116, 225–257.

Christie, Andrew A., 1982, The stochastic behavior of common stock variances: Value, leverage and interest rate effects, *Journal of Financial Economics* 10, 407–432.

Cochrane, John H., 2005, *Asset Pricing* (Princeton University Press: New Jersey) revised edn.

Conrad, Jennifer, Robert Dittmar, and Eric Ghysels, 2008, Skewness and the Bubble, Working paper, University of North Carolina.

Coval, Joshua D., and Tyler Shumway, 2001, Expected option returns, *Journal of Finance* 56, 983–1009.

Cox, John C., and Stephen A. Ross, 1976, The valuation of options for alternative stochastic processes, *Journal of Financial Economics* 3, 145–166.

——— , and Mark Rubinstein, 1979, Option pricing: A simplified approach, *Journal of Financial Economics* 7, 229–263.

Cremers, Martijn, Joost Driessen, Pascal Maenhout, and David Weinbaum, 2004, Individual stock-option prices and credit spreads, Working paper, Yale University.

Cremers, Martijn, and David Weinbaum, 2008, Deviations from put-call parity and stock return predictability, *Journal of Financial and Quantitative Analysis* forthcoming.

Dennis, Patrick, and Stewart Mayhew, 2002, Risk-neutral Skewness: Evidence from stock options, *Journal of Financial and Quantitative Analysis* 37, 471–493.

——— , and Chris Stivers, 2006, Stock returns, implied volatility innovations, and the asymmetric volatility phenomenon, *Journal of Financial and Quantitative Analysis* 41, 381–406.

Driessen, Joost, and Pascal Maenhout, 2007, An empirical portfolio perspective on option pricing anomalies, *Review of Finance* 11, 561–603.

Duan, Jin-Chuan, and Jason Wei, 2007, Systematic risk and the price structure of individual equity options, *Review of Financial Studies* forthcoming.

Duffie, Darrell, Jun Pan, and Kenneth Singleton, 2000, Transform analysis and asset pricing for affine jump-diffusions, *Econometrica* 68, 1343–1376.

Easley, David, Maureen O'Hara, and P. S. Srinivas, 1998, Option volume and stock prices: Evidence on where informed traders trade, *Journal of Finance* 53, 431–465.

Eraker, Bjorn, Michael S. Johannes, and Nicholas Polson, 2003, The impact of jumps in returns and volatility, *Journal of Finance* 58, 1269–1300.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.

———, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Fama, Eugene F., and James D. MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607–636.

Gibbons, Michael R., Stephen A. Ross, and Jay Shanken, 1989, A test of the efficiency of a given portfolio, *Econometrica* 57, 1121–1152.

Goyal, Amit, and Pedro Santa-Clara, 2003, Idiosyncratic risk matters!, *Journal of Finance* 58, 975–1008.

Goyal, Amit, and Alessio Saretto, 2007, Option returns and volatility mispricing, Working paper, Emory University.

Harrison, Michael J., and David M. Kreps, 1979, Martingales and arbitrage in multiperiod securities markets, *Journal of Economic Theory* 20, 381–408.

Harvey, Campbell R., and Akhtar Siddique, 2000, Conditional skewness in asset pricing tests, *Journal of Finance* 55, 1263–1295.

Heston, Steven L., 1993, A closed-form solution for options with stochastic volatility with applications to bond and currency options, *Review of Financial Studies* 6, 327–343.

Jarrow, Robert A., and Eric R. Rosenfeld, 1984, Jump risks and the intertemporal capital asset pricing model, *Journal of Business* 57, 337–351.

Jegadeesh, Narasimhan, 1990, Evidence of predictable behavior of security returns, *Journal of Finance* 45, 881–898.

———, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65–91.

Jiang, George, and Roel Oomen, 2005, A new test of jumps in asset prices, Working paper, University of Arizona.

Jiang, George, and Tong Yao, 2007, The cross-section of stock price jumps and return predictability, Working paper, University of Arizona.

Jorion, Philippe, 1989, On jump processes in the foreign exchange and stock markets, *Review of Financial Studies* 4, 427–445.

Korajczyk, Robert A., and Ronnie Sadka, 2004, Are momentum profits robust to trading costs?, *Journal of Finance* 59, 1039–1082.

Kraus, Alan, and Robert Litzenberger, 1976, Skewness preference and the valuation of risk assets, *Journal of Finance* 31, 1085–1100.

Lakonishok, Josef, Andrei Shleifer, and Robert W. Vishny, 1994, Constrarian investment, extrapolation, and risk, *Journal of Finance* 49, 1541–1578.

Ledoit, Olivier, Pedro Santa-Clara, and Shu Yan, 2003, Relative pricing of options with stochastic volatility , Working paper, UCLA.

Lehmann, Bruce, 1990, Fads, martigales and market efficiency, *Quarterly Journal of Economics* 105, 1–28.

Lesmond, David A., Michael J. Schill, and Chunsheng Zhou, 2003, The illusory nature of momentum profits, *Journal of Financial Economics* 71, 349–380.

Lintner, John, 1965, The valuation of risky assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics* 47, 13–37.

MacBeth, James D., and Larry J. Merville, 1979, An empirical examination of the Black-Scholes call option pricing model, *Journal of Finance* 34, 1173–1186.

Mehra, Rajnish, and Edward C. Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145–161.

Merton, Robert C., 1973, An intertemporal capital asset pricing model, *Econometrica* 41, 867–887.

———, 1976a, Option pricing when the underlying stock returns are discontinuous, *Journal of Financial Economics* 4, 125–144.

———, 1976b, The impact on option pricing of specification error in the underlying stock price returns, *Journal of Finance* 31, 333–350.

———, 1980, On estimating the expected return on the market: An exploratory investigation, *Journal of Financial Economics* 8, 323–361.

Naik, Vasanttilak, and Moon Lee, 1990, General equilibrium pricing of options on the market portfolio with discontinuous returns, *Review of Financial Studies* 3, 493–521.

Ofek, Eli, Matthew Richardson, and Robert Whitelaw, 2004, Limited arbitrage and short sale constraints: Evidence from the option markets, *Journal of Financial Economics* 74, 305–342.

Pan, Jun, 2002, The jump-risk premia implicit in options: Evidence from an integrated time-series study, *Journal of Financial Economics* 63, 3–50.

———, and Allen M. Poteshman, 2006, The information in option volume for future stock prices, *Review of Financial Studies* 19, 871–908.

Press, James S., 1967, A compound events model for security prices, *Journal of Business* 40, 317–335.

Protter, Philip E., 2004, *Stochastic Integration and Differential Equations* (Springer Verlag: New York) 2nd edn.

Rehman, Zahid, and Grigory Vilkov, 2009, Option-based Sentiment for Portfolio Decisions, Working paper, Goethe University.

Rietz, Thomas A., 1988, The equity premium puzzle: A solution?, *Journal of Monetary Economics* 22, 117–131.

Rosenberg, Barr, Kenneth Reid, and Ronald Lanstein, 1985, Persuasive evidence of market infefficiency, *Journal of Portfolio Management* 11, 9–17.

Ross, Stephen, 1978, A simple approach to the valuation of risky streams, *Journal of Business* 51, 453–475.

Rubinstein, Mark, 1976, The valuation of uncertain income streams and the price of options, *Bell Journal of Economics* 7, 407–425.

———, 1985, Nonparametric tests of alternative option pricing models using all reported trades and quotes on the 30 most active CBOE option classes from August 23, 1976 through August 31, 1978, *Journal of Finance* 40, 455–480.

Santa-Clara, Pedro, and Shu Yan, 2008, Crashes, volatility, and the equity premium: Lessons from S&P500 options, *Review of Economics and Statistics* forthcoming.

Sharpe, William, 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425–442.

Spiegel, Matthew, and Xiaotong Wang, 2006, Cross-sectional variation in stock returns: Liquidity and idiosyncratic risk, Working paper, Yale University.

Toft, Klaus B., and Brian Prucyk, 1997, Options on leveraged equity: Theory and empirical tests, *Journal of Finance* 52, 1151–1180.

Zhang, Xiaoyan, Rui Zhao, and Yuhang Xing, 2008, What does individual option volatility smirk tell us about future equity returns?, *Journal of Financial and Quantitative Analysis* forthcoming.

Table 1: **Summary Statistics**

This table reports the summary statistics of the stock and option data during January, 1996 - June, 2005. The individual equity option data are from OptionMetrics. We collect the end-of-month fitted implied volatilities of put and call options ($v_{\text{put}}^{\text{imp}}$ and $v_{\text{call}}^{\text{imp}}$) with one month to expiration and $\Delta = -0.5$ and 0.5 respectively. We define the implied volatility, $v \equiv 0.5(v_{\text{put}}^{\text{imp}} + v_{\text{call}}^{\text{imp}})$, and define the slope of implied volatility smile, $s \equiv v_{\text{put}}^{\text{imp}} - v_{\text{call}}^{\text{imp}}$. We match the option data with the end-of-month stock data obtained from the CRSP and COMPUSTAT. We exclude stocks that do not have two previous years of return data. For each month, we use the market capitalization, book-to-market ratio, and leverage of each stock observed two quarters ago to define the variables ME, BM, and LV, respectively. A stock's β is obtained by regressing its monthly returns on the returns of the S&P 500 index. We present the average and standard deviation across all stocks of ME, BM, LV, β, time series average return r, return skewness, return kurtosis, time series averages of v and s. We also report the correlation coefficients amongst r, v, s, and their changes.

	Mean	Std.
ME ($billion)	3.252	13.108
BM	1.036	5.704
LV	2.024	16.617
β	1.339	1.003
r	0.010	0.060
Skew.	0.408	0.783
Kurt.	4.367	3.083
v	0.567	0.243
s	0.010	0.048
Corr(r, v)	−0.142	0.198
Corr(r, s)	0.076	0.192
Corr(v, s)	−0.011	0.280
Corr$(r, \Delta v)$	−0.305	0.245
Corr$(r, \Delta s)$	0.092	0.203
Corr$(\Delta v, \Delta s)$	−0.020	0.302
Sample length	47	34
Stocks	4048	

Table 2: **Average Skewness of Stock Returns in Slope Quintiles**
We consider the 585 stocks that have implied volatility and slope data during the entire period of January, 1996 - June, 2005. For the i-th stock, let $\{r_t^i\}_{t=1}^T$ denote its monthly return series. Define a *ranking* series $\{I_t^i\}$ so that $I_t^i = n$ if the slope of the stock in month $t-1$ is ranked in the n-th quintile where $n \in (1, ..., 5)$. Fixing a number $n \in (1, ..., 5)$, we collect observations in $\{r_t^i\}_{t=1}^T$ with slope ranking equal to n, that is, $\{r_{t_j}^i : I_{t_j} = n\}$. We then calculate the skewness of the sub-series $\{r_{t_j}^i : I_{t_j} = n\}$. We only consider sub-series of at least ten observations. So we have (at-most) five skewnesses for each stock corresponding to five slope rankings. This table reports the statistics of the skewnesses for all stocks. The last row shows the number of sub-series of stock returns in each quintile ranking.

	Q_1	Q_2	Q_3	Q_4	Q_5
Mean	0.075	0.109	0.187	0.181	0.327
Median	0.778	0.748	0.744	0.751	0.808
Std.	0.062	0.121	0.144	0.157	0.276
Max.	2.802	2.923	2.804	3.557	5.614
Min.	−2.980	−2.627	−2.528	−2.619	−2.095
Obs.	516	541	527	549	491

Table 3: **Returns of Portfolios Formed on Slope of Implied Volatility Smile**
This table reports the statistics for monthly returns of the portfolios formed on s during January, 1996 - June, 2005. Portfolio β is estimated from a market model by running portfolio returns on returns of the S&P 500 index. The data on risk-free interest rate used to compute Sharpe ratio (S.R.) are obtained from Kenneth French's web site. In panel (a), we form quintile portfolios. The last row reports the statistics of the long-short portfolio by long the lowest quintile portfolio and short the highest quintile portfolio. In panel (b), we repeat the exercise for decile portfolios.

(a) Quintile Portfolios

	Mean	t-stat.	β	S.R.	Std.	Skew.	Kurt.	Max.	Min.	Autocorr.
Q_1	0.021	2.802	1.374	0.225	0.080	0.003	3.978	0.294	−0.237	0.115
Q_2	0.013	2.407	1.105	0.175	0.059	−0.608	3.878	0.131	−0.211	0.092
Q_3	0.010	1.972	1.037	0.131	0.055	−0.665	3.475	0.100	−0.181	0.123
Q_4	0.008	1.481	1.085	0.089	0.059	−0.586	3.357	0.127	−0.188	0.112
Q_5	0.002	0.359	1.215	−0.008	0.072	−0.499	3.358	0.142	−0.239	0.132
$Q_1 - Q_5$	0.018	8.168	0.159	0.642	0.024	2.256	13.267	0.161	−0.040	0.053

(b) Decile Portfolios

	Mean	t-stat.	β	S.R.	Std.	Skew.	Kurt.	Max.	Min.	Autocorr.
Q_1	0.023	2.788	1.492	0.228	0.089	0.301	4.727	0.382	−0.248	0.103
Q_2	0.018	2.759	1.255	0.217	0.071	−0.263	3.638	0.206	−0.226	0.119
Q_3	0.014	2.528	1.116	0.188	0.061	−0.470	3.887	0.165	−0.216	0.105
Q_4	0.012	2.243	1.095	0.159	0.058	−0.696	3.901	0.111	−0.207	0.073
Q_5	0.010	1.924	1.056	0.127	0.056	−0.723	3.493	0.101	−0.180	0.134
Q_6	0.010	1.991	1.019	0.133	0.055	−0.562	3.432	0.120	−0.182	0.106
Q_7	0.009	1.689	1.057	0.107	0.058	−0.480	3.345	0.145	−0.185	0.112
Q_8	0.007	1.270	1.112	0.071	0.062	−0.642	3.417	0.112	−0.190	0.111
Q_9	0.004	0.711	1.159	0.022	0.066	−0.623	3.668	0.130	−0.229	0.118
Q_{10}	0.000	0.056	1.274	−0.033	0.079	−0.355	3.168	0.179	−0.249	0.141
$Q_1 - Q_{10}$	0.023	7.777	0.218	0.634	0.031	2.092	11.825	0.203	−0.049	0.051

Table 4: **Time-Series Regressions for Returns of Portfolios Formed on Slope of Implied Volatility Smile**

We run time-series regressions of monthly returns of the quintile portfolios formed on s and the long-short portfolio $Q_1 - Q_5$ on three Fama-French factors ($R_M - R_f$, SMB, and HML) and momentum factor (MOM). We report regression coefficients (t-statistics in parentheses) and R^2.

	Intercept	$R_M - R_f$	SMB	HML	MOM	R^2
Q_1	0.012	1.256	0.863	0.213	−0.249	0.937
	(5.624)	(24.421)	(15.769)	(3.025)	(−4.732)	
Q_2	0.005	1.102	0.510	0.218	−0.021	0.955
	(3.497)	(34.266)	(14.913)	(4.956)	(−0.630)	
Q_3	0.001	1.080	0.449	0.223	0.075	0.952
	(0.811)	(34.996)	(13.668)	(5.288)	(2.383)	
Q_4	−0.001	1.084	0.559	0.189	0.001	0.948
	(−0.462)	(31.124)	(15.090)	(3.964)	(0.042)	
Q_5	−0.008	1.162	0.889	0.212	−0.094	0.952
	(−4.537)	(28.851)	(20.749)	(3.858)	(−2.283)	
$Q_1 - Q_5$	0.019	0.094	−0.026	0.000	−0.155	0.209
	(8.557)	(1.702)	(−0.449)	(0.004)	(−2.745)	

Table 5: **Returns of Portfolios Formed on Control Variables and Slope of Implied Volatility Smile**
This table reports the average monthly returns (t-statistics in parentheses) of the quintile portfolios formed on the control variables and s. The control variables include: market β, past month return r, past month idiosyncratic return r_{idio}, market capitalization ME, book-to-market BM, leverage LV, implied volatility v, idiosyncratic variance v^2_{idio}, historic idiosyncratic volatility $v^{\text{hist}}_{\text{idio}}$, skewness SK, co-skewness CSK, systematic risk proportion v^2_{sys}, option trading volume OV, stock trading volume SV, and stock turn-over TO. We divide stocks into five quintiles by ranking on one of the control variables first and then within each quintile we further divide stocks into five quintiles by ranking on s. The first column of each panel presents the average monthly returns of the quintile portfolios formed on the control variable alone.

	All	Q^s_1	Q^s_2	Q^s_3	Q^s_4	Q^s_5	$Q^s_1 - Q^s_5$
				(a) β			
Q^β_1	0.010	0.020	0.011	0.011	0.006	0.003	0.017
	(2.197)	(3.397)	(2.600)	(2.530)	(1.371)	(0.546)	(6.259)
Q^β_2	0.012	0.020	0.013	0.011	0.009	0.006	0.014
	(2.658)	(3.542)	(3.010)	(2.715)	(2.170)	(1.165)	(5.902)
Q^β_3	0.012	0.022	0.014	0.010	0.011	0.004	0.019
	(2.377)	(3.671)	(2.715)	(2.123)	(2.210)	(0.645)	(7.504)
Q^β_4	0.010	0.020	0.013	0.009	0.007	0.002	0.018
	(1.560)	(2.574)	(1.928)	(1.468)	(1.177)	(0.317)	(6.152)
Q^β_5	0.011	0.021	0.013	0.011	0.009	−0.001	0.022
	(1.016)	(1.746)	(1.264)	(1.112)	(0.821)	(−0.054)	(4.910)
				(b) r			
Q^r_1	0.015	0.015	0.011	0.008	0.008	0.007	0.008
	(1.695)	(4.633)	(3.175)	(2.416)	(2.277)	(2.205)	(6.390)
Q^r_2	0.012	0.017	0.012	0.012	0.011	0.006	0.011
	(2.062)	(3.694)	(2.804)	(2.740)	(2.529)	(1.441)	(5.873)
Q^r_3	0.011	0.023	0.015	0.013	0.010	0.008	0.015
	(2.293)	(4.241)	(2.718)	(2.308)	(1.836)	(1.530)	(6.865)
Q^r_4	0.010	0.024	0.016	0.009	0.008	0.004	0.020
	(1.967)	(2.967)	(1.917)	(1.024)	(0.976)	(0.457)	(7.121)
Q^r_5	0.006	0.019	0.012	0.010	0.002	−0.005	0.024
	(0.882)	(1.506)	(0.967)	(0.864)	(0.181)	(−0.423)	(5.654)
				(c) r_{idio}			
$Q^{r_{\text{idio}}}_1$	0.016	0.025	0.020	0.015	0.016	0.001	0.024
	(1.726)	(2.398)	(2.124)	(1.796)	(1.824)	(0.144)	(6.915)
$Q^{r_{\text{idio}}}_2$	0.014	0.020	0.012	0.013	0.012	0.005	0.015
	(2.350)	(2.803)	(2.238)	(2.188)	(1.991)	(0.739)	(6.452)
$Q^{r_{\text{idio}}}_3$	0.011	0.020	0.014	0.010	0.008	0.005	0.015
	(2.225)	(3.493)	(2.864)	(2.254)	(1.496)	(0.900)	(6.267)
$Q^{r_{\text{idio}}}_4$	0.008	0.019	0.011	0.007	0.008	0.003	0.016
	(1.641)	(3.442)	(2.308)	(1.421)	(1.574)	(0.614)	(6.208)
$Q^{r_{\text{idio}}}_5$	0.007	0.015	0.008	0.006	0.002	−0.000	0.016
	(0.925)	(1.907)	(1.165)	(0.894)	(0.329)	(−0.013)	(5.881)

Table 5 (continued)

	All	Q_1^s	Q_2^s	Q_3^s	Q_4^s	Q_5^s	$Q_1^s - Q_5^s$
				(d) ME			
Q_1^{ME}	0.012	0.018	0.013	0.011	0.008	0.005	0.013
	(1.439)	(2.720)	(2.340)	(2.045)	(1.518)	(0.782)	(5.719)
Q_2^{ME}	0.010	0.017	0.012	0.011	0.010	0.006	0.011
	(1.438)	(3.739)	(3.101)	(2.672)	(2.525)	(1.395)	(6.714)
Q_3^{ME}	0.012	0.022	0.013	0.011	0.009	0.006	0.015
	(1.986)	(3.981)	(2.645)	(2.180)	(1.850)	(1.198)	(5.826)
Q_4^{ME}	0.011	0.022	0.015	0.011	0.011	0.006	0.015
	(2.144)	(2.921)	(2.071)	(1.518)	(1.480)	(0.900)	(5.854)
Q_5^{ME}	0.010	0.020	0.012	0.010	0.002	−0.005	0.025
	(2.166)	(1.720)	(1.007)	(0.913)	(0.155)	(−0.477)	(5.916)
				(e) BM			
Q_1^{BM}	0.009	0.023	0.014	0.012	0.009	0.001	0.022
	(1.141)	(3.161)	(2.517)	(2.510)	(1.597)	(0.127)	(6.875)
Q_2^{BM}	0.010	0.022	0.016	0.010	0.008	0.002	0.020
	(1.480)	(2.971)	(2.852)	(2.029)	(1.632)	(0.276)	(6.150)
Q_3^{BM}	0.011	0.018	0.015	0.010	0.009	0.006	0.012
	(1.791)	(2.503)	(2.607)	(1.956)	(1.622)	(0.830)	(4.058)
Q_4^{BM}	0.012	0.022	0.011	0.009	0.010	0.003	0.019
	(2.151)	(2.562)	(1.744)	(1.569)	(1.652)	(0.406)	(4.873)
Q_5^{BM}	0.012	0.017	0.011	0.009	0.006	0.001	0.016
	(2.213)	(2.062)	(1.672)	(1.385)	(0.820)	(0.133)	(6.251)
				(f) LV			
Q_1^{LV}	0.009	0.025	0.020	0.015	0.017	0.002	0.023
	(0.981)	(2.345)	(2.195)	(1.779)	(1.916)	(0.223)	(6.498)
Q_2^{LV}	0.012	0.022	0.013	0.013	0.012	0.007	0.015
	(1.602)	(3.160)	(2.328)	(2.457)	(2.171)	(1.179)	(5.636)
Q_3^{LV}	0.011	0.021	0.012	0.009	0.009	0.002	0.019
	(1.882)	(3.685)	(2.579)	(2.132)	(1.851)	(0.421)	(7.876)
Q_4^{LV}	0.012	0.015	0.011	0.008	0.005	0.003	0.013
	(2.503)	(2.616)	(2.145)	(1.618)	(1.053)	(0.443)	(5.056)
Q_5^{LV}	0.011	0.018	0.008	0.005	0.002	−0.000	0.018
	(1.967)	(2.194)	(1.238)	(0.820)	(0.294)	(−0.052)	(6.354)

Table 5 (continued)

	All	Q_1^s	Q_2^s	Q_3^s	Q_4^s	Q_5^s	$Q_1^s - Q_5^s$
				(g) v			
Q_1^v	0.010	0.015	0.011	0.008	0.008	0.007	0.008
	(3.020)	(4.633)	(3.175)	(2.416)	(2.277)	(2.205)	(6.390)
Q_2^v	0.011	0.017	0.012	0.012	0.011	0.006	0.011
	(2.745)	(3.694)	(2.804)	(2.740)	(2.529)	(1.441)	(5.873)
Q_3^v	0.014	0.023	0.015	0.013	0.010	0.008	0.015
	(2.608)	(4.241)	(2.718)	(2.308)	(1.836)	(1.530)	(6.865)
Q_4^v	0.012	0.024	0.016	0.009	0.008	0.004	0.020
	(1.501)	(2.967)	(1.917)	(1.024)	(0.976)	(0.457)	(7.121)
Q_5^v	0.008	0.019	0.012	0.010	0.002	−0.005	0.024
	(0.658)	(1.506)	(0.967)	(0.864)	(0.181)	(−0.423)	(5.654)
				(h) v_{idio}^2			
$Q_1^{v_{\text{idio}}^2}$	0.011	0.018	0.013	0.011	0.008	0.005	0.013
	(1.926)	(2.720)	(2.340)	(2.045)	(1.518)	(0.782)	(5.719)
$Q_2^{v_{\text{idio}}^2}$	0.011	0.017	0.012	0.011	0.010	0.006	0.011
	(2.786)	(3.739)	(3.101)	(2.672)	(2.525)	(1.395)	(6.714)
$Q_3^{v_{\text{idio}}^2}$	0.012	0.022	0.013	0.011	0.009	0.006	0.015
	(2.469)	(3.981)	(2.645)	(2.180)	(1.850)	(1.198)	(5.826)
$Q_4^{v_{\text{idio}}^2}$	0.013	0.022	0.015	0.011	0.011	0.006	0.015
	(1.832)	(2.921)	(2.071)	(1.518)	(1.480)	(0.900)	(5.854)
$Q_5^{v_{\text{idio}}^2}$	0.008	0.020	0.012	0.010	0.002	−0.005	0.025
	(0.714)	(1.720)	(1.007)	(0.913)	(0.155)	(−0.477)	(5.916)
				(i) $v_{\text{idio}}^{\text{hist}}$			
$Q_1^{v_{\text{idio}}^{\text{hist}}}$	0.011	0.016	0.012	0.009	0.010	0.008	0.008
	(3.063)	(4.291)	(3.137)	(2.546)	(2.790)	(2.173)	(7.067)
$Q_2^{v_{\text{idio}}^{\text{hist}}}$	0.012	0.018	0.013	0.011	0.010	0.007	0.011
	(2.767)	(3.687)	(3.089)	(2.569)	(2.388)	(1.578)	(5.448)
$Q_3^{v_{\text{idio}}^{\text{hist}}}$	0.012	0.022	0.014	0.012	0.011	0.002	0.020
	(2.227)	(3.545)	(2.523)	(2.174)	(2.053)	(0.369)	(7.567)
$Q_4^{v_{\text{idio}}}$	0.011	0.022	0.014	0.008	0.009	0.001	0.021
	(1.386)	(2.523)	(1.662)	(1.101)	(1.195)	(0.100)	(5.993)
$Q_5^{v_{\text{idio}}^{\text{hist}}}$	0.009	0.021	0.015	0.009	0.001	−0.001	0.022
	(0.799)	(1.638)	(1.352)	(0.834)	(0.074)	(−0.096)	(5.554)

Table 5 (continued)

	All	Q_1^s	Q_2^s	Q_3^s	Q_4^s	Q_5^s	$Q_1^s - Q_5^s$
				(j) SK			
Q_1^{SK}	0.013	0.019	0.014	0.011	0.013	0.007	0.012
	(2.426)	(2.927)	(2.642)	(2.375)	(2.699)	(1.120)	(4.950)
Q_2^{SK}	0.012	0.021	0.012	0.012	0.008	0.005	0.017
	(2.161)	(3.321)	(2.287)	(2.394)	(1.486)	(0.788)	(6.241)
Q_3^{SK}	0.010	0.019	0.013	0.009	0.008	0.001	0.018
	(1.752)	(2.670)	(2.284)	(1.781)	(1.455)	(0.209)	(6.107)
Q_4^{SK}	0.011	0.024	0.013	0.011	0.007	0.002	0.022
	(1.841)	(3.070)	(2.168)	(2.010)	(1.211)	(0.298)	(6.824)
Q_5^{SK}	0.009	0.022	0.012	0.009	0.004	−0.001	0.023
	(1.137)	(2.159)	(1.577)	(1.347)	(0.540)	(−0.161)	(5.767)
				(k) CSK			
Q_1^{CSK}	0.010	0.020	0.011	0.011	0.006	0.003	0.017
	(2.197)	(3.397)	(2.600)	(2.530)	(1.371)	(0.546)	(6.259)
Q_2^{CSK}	0.012	0.020	0.013	0.011	0.009	0.006	0.014
	(2.658)	(3.542)	(3.010)	(2.715)	(2.170)	(1.165)	(5.902)
Q_3^{CSK}	0.012	0.022	0.014	0.010	0.011	0.004	0.019
	(2.377)	(3.671)	(2.715)	(2.123)	(2.210)	(0.645)	(7.504)
Q_4^{CSK}	0.010	0.020	0.013	0.009	0.007	0.002	0.018
	(1.560)	(2.574)	(1.928)	(1.468)	(1.177)	(0.317)	(6.152)
Q_5^{CSK}	0.011	0.021	0.013	0.011	0.009	−0.001	0.022
	(1.016)	(1.746)	(1.264)	(1.112)	(0.821)	(−0.054)	(4.910)
				(l) v_{sys}^2			
$Q_1^{v_{\mathrm{sys}}^2}$	0.012	0.023	0.014	0.012	0.009	0.001	0.022
	(2.059)	(3.161)	(2.517)	(2.510)	(1.597)	(0.127)	(6.875)
$Q_2^{v_{\mathrm{sys}}^2}$	0.012	0.022	0.016	0.010	0.008	0.002	0.020
	(2.081)	(2.971)	(2.852)	(2.029)	(1.632)	(0.276)	(6.150)
$Q_3^{v_{\mathrm{sys}}^2}$	0.012	0.018	0.015	0.010	0.009	0.006	0.012
	(1.968)	(2.503)	(2.607)	(1.956)	(1.622)	(0.830)	(4.058)
$Q_4^{v_{\mathrm{sys}}^2}$	0.011	0.022	0.011	0.009	0.010	0.003	0.019
	(1.681)	(2.562)	(1.744)	(1.569)	(1.652)	(0.406)	(4.873)
$Q_5^{v_{\mathrm{sys}}^2}$	0.009	0.017	0.011	0.009	0.006	0.001	0.016
	(1.235)	(2.062)	(1.672)	(1.385)	(0.820)	(0.133)	(6.251)

Table 5 (continued)

	All	Q_1^s	Q_2^s	Q_3^s	Q_4^s	Q_5^s	$Q_1^s - Q_5^s$
				(m) OV			
Q_1^{OV}	0.013	0.020	0.017	0.011	0.012	0.004	0.016
	(2.400)	(3.046)	(3.456)	(2.270)	(2.243)	(0.667)	(6.311)
Q_2^{OV}	0.011	0.022	0.014	0.010	0.008	0.002	0.021
	(2.071)	(3.216)	(2.693)	(2.038)	(1.531)	(0.313)	(6.980)
Q_3^{OV}	0.012	0.020	0.016	0.011	0.009	0.005	0.015
	(1.991)	(2.504)	(2.764)	(2.121)	(1.587)	(0.690)	(5.053)
Q_4^{OV}	0.012	0.025	0.013	0.010	0.008	0.001	0.024
	(1.698)	(2.799)	(1.975)	(1.844)	(1.366)	(0.130)	(6.267)
Q_5^{OV}	0.007	0.012	0.010	0.010	0.009	−0.004	0.016
	(1.023)	(1.423)	(1.529)	(1.497)	(1.261	(−0.488)	(4.539)
				(n) SV			
SV	All	Q_1^s	Q_2^s	Q_3^s	Q_4^s	Q_5^s	$Q_1^s - Q_5^s$
All		0.021	0.013	0.010	0.008	0.002	0.018
		(2.802)	(2.407)	(1.972)	(1.481)	(0.359)	(8.168)
Q_1^{SV}	0.011	0.021	0.016	0.009	0.008	0.001	0.020
	(1.998)	(3.164)	(2.994)	(1.881)	(1.436)	(0.084)	(7.735)
Q_2^{SV}	0.013	0.022	0.016	0.012	0.010	0.003	0.019
	(2.137)	(3.001)	(2.830)	(2.229)	(1.828)	(0.435)	(5.967)
Q_3^{SV}	0.011	0.023	0.012	0.009	0.008	0.002	0.021
	(1.772)	(2.777)	(2.053)	(1.917)	(1.338)	(0.261)	(5.781)
Q_4^{SV}	0.011	0.019	0.014	0.012	0.008	0.004	0.015
	(1.859)	(2.410)	(2.340)	(2.287)	(1.486)	(0.529)	(4.381)
Q_5^{SV}	0.010	0.016	0.010	0.010	0.009	0.003	0.014
	(1.346)	(1.787)	(1.545)	(1.645)	(1.319)	(0.327)	(4.105)
				(o) TO			
Q_1^{TO}	0.010	0.018	0.013	0.009	0.008	0.001	0.016
	(2.315)	(3.303)	(3.223)	(2.369)	(1.945)	(0.303)	(7.304)
Q_2^{TO}	0.012	0.023	0.013	0.011	0.009	0.006	0.017
	(2.603)	(3.981)	(2.820)	(2.709)	(1.888)	(1.037)	(7.552)
Q_3^{TO}	0.011	0.020	0.014	0.010	0.008	0.002	0.018
	(1.930)	(2.960)	(2.474)	(1.878)	(1.619)	(0.351)	(6.887)
Q_4^{TO}	0.013	0.025	0.013	0.011	0.010	0.004	0.020
	(1.770)	(2.609)	(2.026)	(1.742)	(1.467)	(0.523)	(5.380)
Q_5^{TO}	0.009	0.019	0.012	0.012	0.006	−0.001	0.020
	(0.931)	(1.622)	(1.172)	(1.209)	(0.576	(−0.059)	(4.876)

Table 6: **Fama-MacBeth Regressions**

This table reports the estimated coefficients (t-statistics in parentheses) of Fama-MacBeth regressions for monthly stock returns. In addition to s, the explanatory variables include β, lagged return r, log size ln(ME), book-to-market BM, leverage LV, implied volatility v, skewness SK, stock volume SV, option volume OV, and stock turnover TO.

s	−0.057	−0.061	−0.056	−0.055	−0.059	−0.060	−0.054	−0.057	−0.057	−0.057	−0.057	−0.057
	(−9.804)	(−10.552)	(−9.847)	(−10.036)	(−9.479)	(−9.645)	(−9.580)	(−10.039)	(−9.840)	(−9.859)	(−9.983)	(−9.531)
β		0.001										0.000
		(0.414)										(0.121)
r			−0.007									−0.025
			(−0.491)									(−2.964)
ln(ME)				−0.000								−0.001
				(−0.129)								(−1.681)
BM					−0.133							0.332
					(−0.712)							(0.854)
LV						−0.000						−0.000
						(−0.506)						(−1.354)
v							−0.009					−0.013
							(−0.582)					(−1.047)
SK								−0.002				−0.001
								(−1.007)				(−0.975)
SV									−0.000			−0.000
									(−0.156)			(−0.582)
OV										0.000		0.000
										(0.784)		(1.395)
TO											0.000	0.000
											(0.564)	(0.586)

Table 7: **Returns of Portfolios Formed on Slope of Implied Volatility Smile by Calendar Months**

This table reports the average monthly returns (t-statistics in parentheses) of the quintile portfolios formed on s by calendar months. The last column reports the average monthly returns of the long-short portfolio by long the lowest quintile portfolio and short the highest quintile portfolio.

	Q_1	Q_2	Q_3	Q_4	Q_5	$Q_1 - Q_5$
Jan.	0.047	0.014	0.003	0.004	0.010	0.037
	(1.396)	(0.843)	(0.273)	(0.351)	(0.538)	(2.172)
Feb.	−0.004	0.004	0.004	−0.005	−0.010	0.006
	(−0.181)	(0.217)	(0.228)	(−0.271)	(−0.434)	(0.930)
Mar.	0.011	0.010	0.006	0.004	−0.004	0.016
	(0.654)	(0.636)	(0.459)	(0.284)	(−0.243)	(2.605)
Apr.	0.030	0.022	0.019	0.022	0.014	0.016
	(1.054)	(1.150)	(1.051)	(1.011)	(0.553)	(3.712)
May	0.044	0.028	0.021	0.020	0.028	0.016
	(1.702)	(1.629)	(1.274)	(1.140)	(1.229)	(2.978)
June	0.020	0.010	0.012	0.012	0.009	0.010
	(1.018)	(0.692)	(0.932)	(0.920)	(0.572)	(2.535)
July	−0.017	−0.014	−0.019	−0.029	−0.049	0.032
	(−0.684)	(−0.677)	(−0.976)	(−1.372)	(−1.798)	(3.321)
Aug.	−0.003	−0.009	−0.008	−0.011	−0.015	0.013
	(−0.075)	(−0.309)	(−0.295)	(−0.422)	(−0.487)	(2.777)
Sept.	−0.004	−0.007	−0.007	−0.012	−0.017	0.013
	(−0.130)	(−0.289)	(−0.307)	(−0.443)	(−0.560)	(1.534)
Oct.	0.037	0.032	0.027	0.024	0.014	0.023
	(1.529)	(1.996)	(1.739)	(1.364)	(0.674)	(3.738)
Nov.	0.063	0.042	0.036	0.044	0.040	0.023
	(1.971)	(1.976)	(1.697)	(2.053)	(1.548)	(3.184)
Dec.	0.031	0.031	0.030	0.028	0.012	0.019
	(1.902)	(2.100)	(2.243)	(1.885)	(0.724)	(3.257)

Table 8: **Returns of Portfolios Formed on Slope of Implied Volatility Smile for Subsamples of Stocks**

This table reports average monthly returns (t-statistics in parentheses) of the quintile portfolios formed on s for subsamples of stocks obtained by imposing various filters. First, we exclude stocks with outliers of s, that is, a stock is not used to form quintile portfolios in a particular month if $s < -0.2$ or $s > 0.2$. Second, we exclude financial firms from the sample. Third, we only use stocks that have observations of s for the full sample period. Fourth, we only use stocks that paid dividends during the sample period. Last, we use stocks that did not pay dividends during the sample period.

	Q_1	Q_2	Q_3	Q_4	Q_5	$Q_1 - Q_5$
Excluding outliers	0.020	0.013	0.010	0.008	0.004	0.016
	(2.795)	(2.315)	(2.006)	(1.508)	(0.642)	(8.194)
Excluding financial firms	0.021	0.013	0.010	0.008	0.002	0.019
	(2.662)	(2.226)	(1.809)	(1.303)	(0.292)	(8.025)
Stocks with full sample	0.026	0.016	0.012	0.012	0.013	0.012
	(4.066)	(3.354)	(2.538)	(2.655)	(2.456)	(5.541)
Dividend-paying stocks	0.022	0.013	0.011	0.010	0.006	0.016
	(3.271)	(2.552)	(2.342)	(1.912)	(0.982)	(7.937)
Non-dividend-paying stocks	0.021	0.010	0.004	−0.005	−0.006	0.027
	(2.032)	(1.069)	(0.501)	(−0.648)	(−0.648)	(6.106)

Table 9: **Different Holding Period Returns of Portfolios Formed on Slope of Implied Volatility Smile**

This table reports the average monthly returns (t-statistics in parentheses) of the quintile portfolios formed on s for different holding periods. The last column reports the average monthly returns of the long-short portfolio by long the lowest quintile portfolio and short the highest quintile portfolio. For horizons longer than one month, we use the Newey-West procedure to compute the t-statistics because the returns are serially correlated by construction.

	Q_1	Q_2	Q_3	Q_4	Q_5	$Q_1 - Q_5$
1 month	0.021	0.013	0.010	0.008	0.002	0.018
	(2.802)	(2.407)	(1.972)	(1.481)	(0.359)	(8.168)
2 months	0.017	0.013	0.010	0.010	0.007	0.010
	(2.665)	(2.314)	(1.869)	(1.410)	(0.339)	(2.574)
3 months	0.016	0.013	0.011	0.010	0.010	0.006
	(2.790)	(2.384)	(1.908)	(1.454)	(0.344)	(2.536)
4 months	0.016	0.013	0.011	0.011	0.011	0.005
	(2.933)	(2.470)	(1.970)	(1.518)	(0.356)	(2.625)
5 months	0.015	0.013	0.011	0.012	0.011	0.004
	(3.135)	(2.614)	(2.070)	(1.610)	(0.376)	(2.757)
6 months	0.015	0.013	0.012	0.012	0.012	0.004
	(3.301)	(2.754)	(2.169)	(1.697)	(0.393)	(2.874)

Table 10: **Different Measures of Slope of Implied Volatility Smile**

Panel (a) reports the average monthly returns (t-statistics in parentheses) of the quintile portfolios formed on different measures of slope of implied volatility smile. The last column reports the average monthly returns of the long-short portfolio by long the lowest quintile portfolio and short the highest quintile portfolio. These measures other than s are defined as: $\hat{s} \equiv \left(\sigma_{\text{put}}^{\text{imp}} - \sigma_{\text{call}}^{\text{imp}}\right)/v$, $sk = \sigma_{\text{put}}^{\text{imp}}(-0.25) - \sigma_{\text{call}}^{\text{imp}}$, $sl = \sigma_{\text{put}}^{\text{imp}}(-0.25) - \sigma_{\text{put}}^{\text{imp}}$, where $\sigma_{\text{put}}^{\text{imp}}(-0.25)$ is the implied volatility of the put option with $\Delta = -0.25$. Panel (b) reports the summary statistics of these different measures.

(a) Portfolio Returns

	Q_1	Q_2	Q_3	Q_4	Q_5	$Q_1 - Q_5$
s	0.021	0.013	0.010	0.008	0.002	0.018
	(2.802)	(2.407)	(1.972)	(1.481)	(0.359)	(8.168)
$\hat{s}$	0.021	0.013	0.010	0.007	0.004	0.017
	(3.199)	(2.000)	(1.642)	(1.232)	(0.687)	(7.118)
sk	0.020	0.013	0.010	0.008	0.004	0.015
	(2.641)	(2.360)	(1.872)	(1.436)	(0.657)	(7.386)
sl	0.013	0.013	0.011	0.011	0.007	0.006
	(1.896)	(2.331)	(1.894)	(1.873)	(1.100)	(3.246)

(b) Summary Statistics

			Correlations		
	Mean	Std.	$\hat{s}$	sk	sl
s	0.010	0.086	0.965	0.792	0.283
$\hat{s}$	0.022	0.136		0.775	0.291
sk	0.024	0.127			0.768
sl	0.014	0.070			

Table 11: **Double Sort on s, sk, and sl**

This table reports the average monthly returns (t-statistics in parentheses) of double-sorted quintile portfolios formed on s, sk, and sl. In panels (a) and (b), we sort on sk and sl first and then within each quintile we sort on s. In panels (c) and (d), we sort on s first and then within each quintile we sort on sk and sl.

(a)

sk	Q_1^s	Q_2^s	Q_3^s	Q_4^s	Q_5^s	$Q_1^s - Q_5^s$
Q_1^{sk}	0.026	0.023	0.021	0.018	0.010	0.016
	(2.786)	(2.884)	(2.988)	(2.670)	(1.380)	(3.296)
Q_2^{sk}	0.018	0.015	0.011	0.012	0.010	0.008
	(2.599)	(2.795)	(2.176)	(2.233)	(1.609)	(2.681)
Q_3^{sk}	0.014	0.010	0.010	0.008	0.008	0.006
	(2.348)	(1.967)	(2.038)	(1.490)	(1.243)	(2.502)
Q_4^{sk}	0.010	0.008	0.011	0.007	0.004	0.006
	(1.667)	(1.612)	(2.111)	(1.209)	(0.540)	(2.322)
Q_5^{sk}	0.011	0.006	0.005	0.003	−0.004	0.016
	(1.825)	(0.996)	(0.828)	(0.453)	(−0.542)	(4.348)

(b)

sl	Q_1^s	Q_2^s	Q_3^s	Q_4^s	Q_5^s	$Q_1^s - Q_5^s$
Q_1^{sl}	0.022	0.018	0.011	0.011	0.003	0.019
	(2.744)	(2.556)	(1.648)	(1.673)	(0.464)	(5.931)
Q_2^{sl}	0.022	0.011	0.013	0.011	0.007	0.016
	(3.508)	(2.150)	(2.490)	(2.015)	(1.053)	(5.713)
Q_3^{sl}	0.019	0.015	0.010	0.008	0.004	0.015
	(2.300)	(2.675)	(1.998)	(1.540)	(0.520)	(3.655)
Q_4^{sl}	0.022	0.012	0.011	0.008	0.004	0.018
	(2.759)	(2.177)	(2.036)	(1.514)	(0.516)	(5.591)
Q_5^{sl}	0.014	0.010	0.008	0.005	−0.003	0.018
	(1.927)	(1.855)	(1.371)	(0.814)	(−0.471)	(5.297)

(c)

s	Q_1^{sk}	Q_2^{sk}	Q_3^{sk}	Q_4^{sk}	Q_5^{sk}	$Q_1^{sk} - Q_5^{sk}$
Q_1^s	0.024	0.022	0.023	0.021	0.015	0.009
	(2.665)	(2.766)	(3.122)	(3.147)	(2.006)	(2.722)
Q_2^s	0.015	0.012	0.013	0.015	0.012	0.003
	(2.204)	(2.210)	(2.583)	(2.617)	(2.103)	(0.971)
Q_3^s	0.011	0.011	0.010	0.009	0.011	0.000
	(1.699)	(2.185)	(1.883)	(1.873)	(1.918)	(0.117)
Q_4^s	0.008	0.007	0.010	0.007	0.009	−0.000
	(1.263)	(1.270)	(1.796)	(1.370)	(1.491)	(−0.081)
Q_5^s	0.005	0.002	0.005	−0.000	−0.001	0.006
	(0.793)	(0.355)	(0.767)	(−0.009)	(−0.129)	(1.994)

(d)

s	Q_1^{sl}	Q_2^{sl}	Q_3^{sl}	Q_4^{sl}	Q_5^{sl}	$Q_1^{sl} - Q_5^{sl}$
Q_1^s	0.019	0.023	0.022	0.023	0.018	0.001
	(2.436)	(3.214)	(3.110)	(2.624)	(2.286)	(0.599)
Q_2^s	(0.013	0.014	0.014	0.014	0.012	0.000
	(1.862)	(2.533)	(2.743)	(2.435)	(2.132)	(0.093)
Q_3^s	0.011	0.011	0.008	0.010	0.011	0.000
	(1.654)	(2.188)	(1.653)	(2.056)	(1.960)	(0.002)
Q_4^s	0.008	0.008	0.008	0.008	0.009	−0.001
	(1.241)	(1.379)	(1.420)	(1.546)	(1.613)	(−0.288)
Q_5^s	0.003	0.002	0.004	0.002	0.001	0.002
	(0.369)	(0.253)	(0.614)	(0.343)	(0.146)	(0.526)

Figure 1: **Average Slopes and Returns of Quintile Portfolios**
Panel (a) plots monthly average slopes of the quintile portfolios formed on s during January, 1996 - June, 2005 while panel (b) plots monthly returns of these portfolios during February, 1996 - July, 2005. Panel (c) plots returns of the long-short portfolio $Q_1 - Q_5$ during February, 1996 - July, 2005.